ADVANCED SOIL MECHANICS

By : G. Habibagahi

Course Outline

- Review of classical Soil mechanics
- Elasticity in soil mechanics
- Plasticity
- Elasto-plastic models (Modified Cam-clay)
- Critical State
- Shear Strength
- Hyperbolic model
- 3D consolidation -Biot Theory
  -Mandel cryer's effect
  -Constant Gradient
- Generalized slope stability

Course Evaluation & texts

Course Evaluation	
Homeworks	25%
Presentation	25%
Final Exam	50%

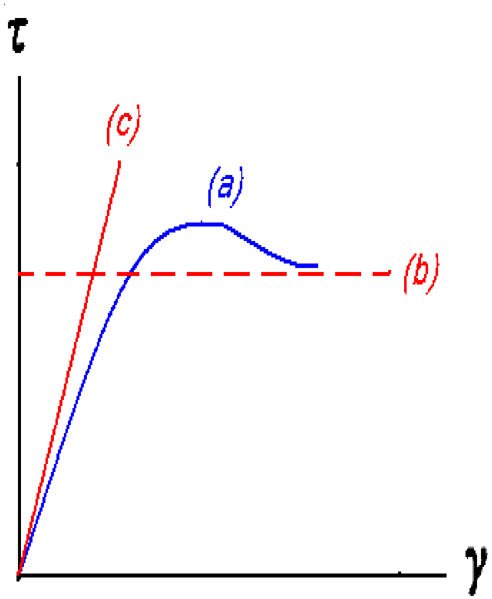
Texts

- Soil Behavior and Critical State Soil Mechanics(D.M. Wood)
- Plasticity and Soil mechanics(R.O.Davis & A.P.S. Selvadurai)
- Geotechnical Modeling(D.M. Wood)

Chapter #1

Soil Modeling

Soil Modeling



a) : Real Soil Behavior

b) : Rigid-perfectly Plastic

(stability calculation, bearing capacity)

c) : Linear Elastic

(settlement calculation)

Soil Modeling

- These two simple models lie behind much of "classical soil mechanics".

- Therefore, there is usually a need to determine the "stiffness" and strength of the soil.

- These are the "elementary models".

- There are other models although having certain assumptions which are more realistic.

Soil Modeling

- A good model should be able to relate various aspects of soil behavior such as :

  -Strength

  -Compression

  -Dilatancy

  -Critical States

  In order to be capable of predicting real soil response.

Soil Modeling

- Some notations based on triaxial cell data :

$$\sigma_a = \sigma_r \left(1 - \frac{a}{A}\right) + \frac{F}{A}$$

$a : Area\,of\,loading\,ram$

$\frac{a}{A} = Negligible$

→

Deviator stress

$$q = \sigma_a - \sigma_r \cong \frac{F}{A}$$

$$\sigma_a = \sigma_r + \frac{F}{A}$$

$$\sigma'_a = \sigma_a - u$$

$$\sigma'_r = \sigma_r - u$$

→

$$q = \sigma'_a - \sigma'_r$$

Soil Modeling

- Mean stress

$$p = \frac{\sigma_a + 2\sigma_r}{3}$$

- Deviator stress

$$q = \sigma'_a - \sigma'_r$$

- Volumetric strain

$$\delta\varepsilon_p = -\frac{\delta v}{v}$$

Soil Modeling

- The work input for a cubical element under general state of stress per unit volume of the element is :

$$dw = \sigma'_{xx}\delta\varepsilon_{xx} + \sigma'_{yy}\delta\varepsilon_{yy} + \sigma'_{zz}\delta\varepsilon_{zz} +$$

$$\tau_{yz}\delta\gamma_{yz} + \tau_{zx}\delta\gamma_{zx} + \tau_{xy}\delta\gamma_{xy}$$

- If soil element's stress are considered in the principal directions :

$$dw = \sigma'_1\delta\varepsilon_1 + \sigma'_2\delta\varepsilon_2 + \sigma'_3\delta\varepsilon_3$$

Soil Modeling

- For triaxial specimen

$$\sigma'_r = \sigma'_2 = \sigma'_3$$
$$\sigma'_a = \sigma'_1$$

$$\delta\varepsilon_r = \delta\varepsilon_2 = \delta\varepsilon_3$$
$$\delta\varepsilon_a = \delta\varepsilon_1$$

$$dw = \sigma'_a \delta\varepsilon_a + 2\sigma'_r \delta\varepsilon_r$$

$$\delta\varepsilon_p = \delta\varepsilon_a + 2\delta\varepsilon_r$$

Soil Modeling

- Increment of volumetric work :

$$\delta w_v = p' \delta \varepsilon_p \qquad p' = \frac{\sigma'_a + 2\sigma'_r}{3}$$

- Increment of distortional work :

$$\delta w_d = q \delta \varepsilon_q \qquad \delta \varepsilon_q = \frac{2}{3}(\delta \varepsilon_a - \delta \varepsilon_r)$$

Therefore :

$$\delta w = p' \delta \varepsilon_p + q \delta \varepsilon_q$$

$$= \delta w_v + \delta w_d$$

Soil Modeling

- In General : (first & second invariants of the tensor)

$$p' = \frac{\sigma'_{xx} + \sigma'_{yy} + \sigma'_{zz}}{3}$$

$$q = \left[\frac{(\sigma'_{yy} - \sigma'_{zz})^2 + (\sigma'_{zz} - \sigma'_{xx})^2 + (\sigma'_{xx} - \sigma'_{yy})^2}{2} + 3(\tau_{yz}^2 + \tau_{zx}^2 + \tau_{xy}^2) \right]^{1/2}$$

$$\delta\varepsilon_p = \delta\varepsilon_{xx} + \delta\varepsilon_{yy} + \delta\varepsilon_{zz}$$

$$\delta\varepsilon_q = \tfrac{1}{3}\{2[(\delta\varepsilon_{yy} - \delta\varepsilon_{zz})^2 + (\delta\varepsilon_{zz} - \delta\varepsilon_{xx})^2 + (\delta\varepsilon_{xx} - \delta\varepsilon_{yy})^2] + 3(\delta\gamma_{yz}^2 + \delta\gamma_{zx}^2 + \delta\gamma_{xy}^2)\}^{1/2}$$

Soil Modeling

- For deformations at constant volume :

$$\delta\varepsilon_p = 0 = \delta\varepsilon_a + 2\delta\varepsilon_r$$

$$\Rightarrow \quad \delta\varepsilon_q = \frac{2}{3}(\delta\varepsilon_a - \delta\varepsilon_r) = -2\delta\varepsilon_r = \delta\varepsilon_a$$

$$\Rightarrow \quad \boxed{\delta\varepsilon_q = \delta\varepsilon_a}$$

at constant volume change

Soil Modeling

- From triaxial test data :

$$\delta\varepsilon_a = -\frac{\delta l}{l}$$

$$\delta\varepsilon_p = -\frac{\delta v}{v}$$

$$\delta\varepsilon_r = \frac{-\frac{\delta v}{v} + \frac{\delta l}{l}}{2}$$

$$\delta\varepsilon_q = \frac{2}{3}\left(\delta\varepsilon_a - \delta\varepsilon_r\right)$$

$$= \frac{2}{3}\left(-\frac{\delta l}{l} + \frac{1}{2}\left(\frac{\delta v}{v} - \frac{\delta l}{l}\right)\right)$$

$$\delta\varepsilon_q = \frac{1}{3}\frac{\delta v}{v} - \frac{\delta l}{l}$$

Soil Modeling

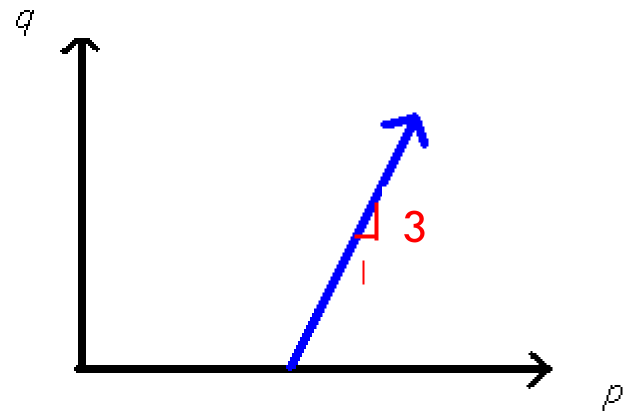
$$p = \frac{\sigma_a + 2\sigma_r}{3}$$

$$q = \sigma_a - \sigma_r$$

$$p = \sigma_r + \frac{1}{3}q$$

$$if \quad \sigma_r \quad is \quad held \quad const. \qquad \delta\sigma_r = 0$$

$$\delta q = 3\delta p$$



Conventional compression test

Soil Modeling

- Undrained Test :

  -Constant Mass Test

→ Constant Volume Test ; if all the soil components are incompressible

Chapter #2

Elasticity

Elasticity

- E : Young's Modulus $\quad\quad$ ν : Poisson's Ratio

$$E = \frac{P/A}{\delta l/l}$$

$$\nu = \frac{-\delta d/d}{\delta l/l}$$

- A pair of elastic parameters is sufficient to describe the elastic response of isotropic materials.
- other pairs of elastic parameters often used :

  G : Shear Modulus

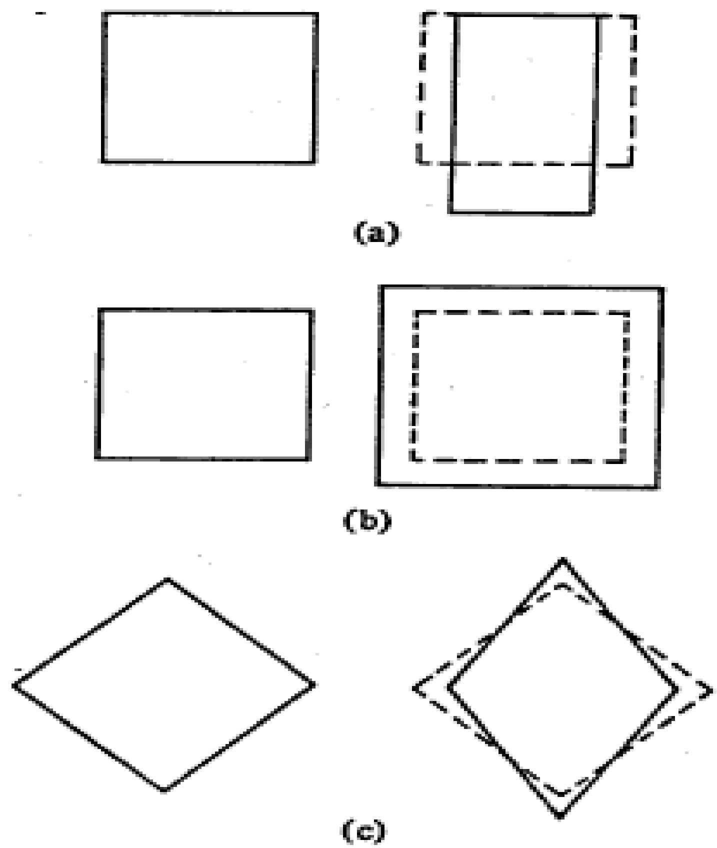
  K : Bulk Modulus

Elasticity

- In this way elastic deformation is devided into two components :

  -Volumetric part

  -Distortional part



Hooke's law

axial symmetry:

$$\begin{pmatrix} \delta\varepsilon_a \\ \delta\varepsilon_r \end{pmatrix} = \frac{1}{E}\begin{pmatrix} 1 & -2\nu \\ -\nu & 1-\nu \end{pmatrix}\begin{pmatrix} \delta\sigma'_a \\ \delta\sigma'_r \end{pmatrix}$$

$$\begin{pmatrix} \delta\sigma'_a \\ \delta\sigma'_r \end{pmatrix} = \frac{E}{(1+\nu)(1-2\nu)}\begin{pmatrix} 1-\nu & 2\nu \\ \nu & 1 \end{pmatrix}\begin{pmatrix} \delta\varepsilon_a \\ \delta\varepsilon_r \end{pmatrix}$$

compliance/stiffness matrices not symmetric because stress and strain variables not work conjugate

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Elasticity

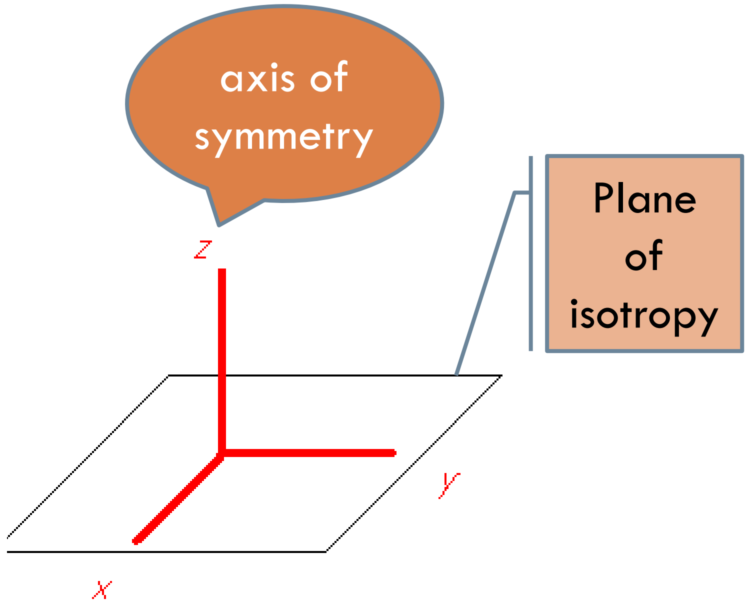
- In general (for isotropic materials) :

$$\begin{bmatrix}\delta\sigma'_{xx}\\ \delta\sigma'_{yy}\\ \delta\sigma'_{zz}\\ \delta\sigma_{yz}\\ \delta\sigma_{zx}\\ \delta\sigma_{xy}\end{bmatrix}=\begin{bmatrix}K & K & K & 0 & 0 & 0\\ K & K & K & 0 & 0 & 0\\ K & K & K & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\delta\varepsilon_{xx}\\ \delta\varepsilon_{yy}\\ \delta\varepsilon_{zz}\\ \delta\gamma_{yz}\\ \delta\gamma_{zx}\\ \delta\gamma_{xy}\end{bmatrix}+\begin{bmatrix}\frac{4G}{3} & -\frac{2G}{3} & -\frac{2G}{3} & 0 & 0 & 0\\ -\frac{2G}{3} & \frac{4G}{3} & -\frac{2G}{3} & 0 & 0 & 0\\ -\frac{2G}{3} & -\frac{2G}{3} & \frac{4G}{3} & 0 & 0 & 0\\ 0 & 0 & 0 & G & 0 & 0\\ 0 & 0 & 0 & 0 & G & 0\\ 0 & 0 & 0 & 0 & 0 & G\end{bmatrix}\begin{bmatrix}\delta\varepsilon_{XX}\\ \delta\varepsilon_{YY}\\ \delta\varepsilon_{ZZ}\\ \delta\gamma_{YZ}\\ \delta\gamma_{ZX}\\ \delta\gamma_{XY}\end{bmatrix}$$

Elasticity

- For transversly isotropic material :

$$
\begin{bmatrix} \varepsilon_X \\ \varepsilon_Y \\ \varepsilon_Z \\ \gamma_{XY} \\ \gamma_{YZ} \\ \gamma_{ZX} \end{bmatrix}
=
\begin{bmatrix}
1/E & -\frac{\vartheta}{E} & -\frac{\vartheta'}{E'} & 0 & 0 & 0 \\
-\frac{\vartheta}{E} & \frac{1}{E} & -\frac{\vartheta'}{E'} & 0 & 0 & 0 \\
-\frac{\vartheta'}{E'} & -\frac{\vartheta'}{E'} & \frac{1}{E} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G'} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G'}
\end{bmatrix}
\begin{bmatrix} \sigma_X \\ \sigma_Y \\ \sigma_Z \\ \tau_{XY} \\ \tau_{YZ} \\ \tau_{ZX} \end{bmatrix}
$$



Elasticity

- Five Constants :

 -E

 -E'

 - ν

 - ν'

 -G'

Assignment # 1

a. Write the transversly isotropic equation in 2-D form for a triaxial specimen and find the matrix [C] :

$$\begin{Bmatrix} \delta p' \\ \delta q \end{Bmatrix} = [C] \begin{Bmatrix} \delta \varepsilon_p \\ \delta \varepsilon_q \end{Bmatrix}$$

Is volumetric and distortional effects independent?

Elasticity

- For plane strain :

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} K + \frac{4G}{3} & K - \frac{2G}{3} & 0 \\ K - \frac{2G}{3} & K + \frac{4G}{3} & 0 \\ 0 & 0 & G \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}$$

$$K = \frac{E}{3(1-2\vartheta)} \qquad G = \frac{E}{2(1+\vartheta)}$$

Elasticity

- For triaxial test : ($\sigma_{xy} = 0$ $\sigma_{xx} = \sigma_3 = \sigma_2$ $\sigma_{yy} = \sigma_1$)

$$\delta\varepsilon_p = \delta\varepsilon_a + 2\delta\varepsilon_r$$

$$\delta\varepsilon_q = \frac{2}{3}(\delta\varepsilon_a - \delta\varepsilon_r)$$

$$\begin{Bmatrix} \delta\varepsilon_p \\ \delta\varepsilon_q \end{Bmatrix} = \begin{bmatrix} \frac{1}{K'} & 0 \\ 0 & \frac{1}{3G'} \end{bmatrix} \begin{Bmatrix} \delta p' \\ \delta q \end{Bmatrix}$$

K' & G' : Effective Elastic parameters

Elasticity

Or

$$\begin{Bmatrix} \delta p' \\ \delta q \end{Bmatrix} = \begin{bmatrix} K' & 0 \\ 0 & 3G' \end{bmatrix} \begin{Bmatrix} \delta \varepsilon_p \\ \delta \varepsilon_q \end{Bmatrix} \qquad \text{(A)}$$

Since off-diagonal terms are zero ; there is no coupling between volumetric and distortional effects for isotropic elastic behavior.

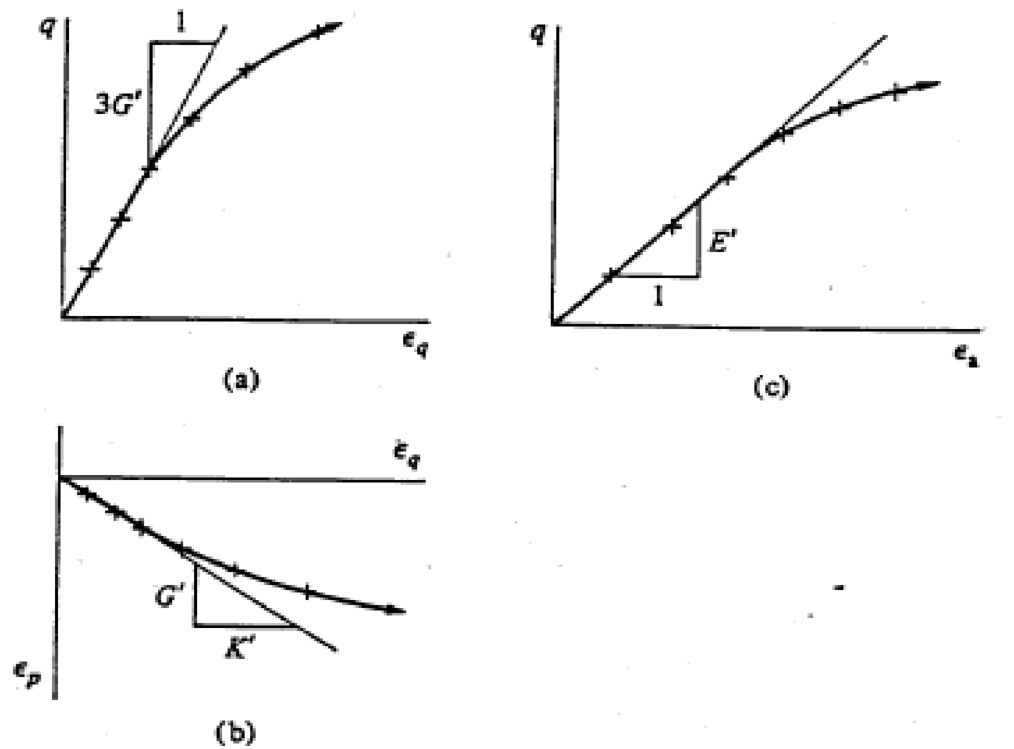
$\delta p'$ → no distortional effect

δq → no change in volume

Elasticity

- For Drained condition :
- $$\frac{\delta\varepsilon_p}{\delta\varepsilon_q} = (\frac{3G'}{K'})(\frac{\delta p'}{\delta q})$$, in triaxial test : $$\delta q = 3\delta p'$$

$$\frac{\delta\varepsilon_p}{\delta\varepsilon_q} = \frac{G'}{K'}$$



Elasticity

- For Undrained condition :

No volume change $\delta\varepsilon_p = 0$

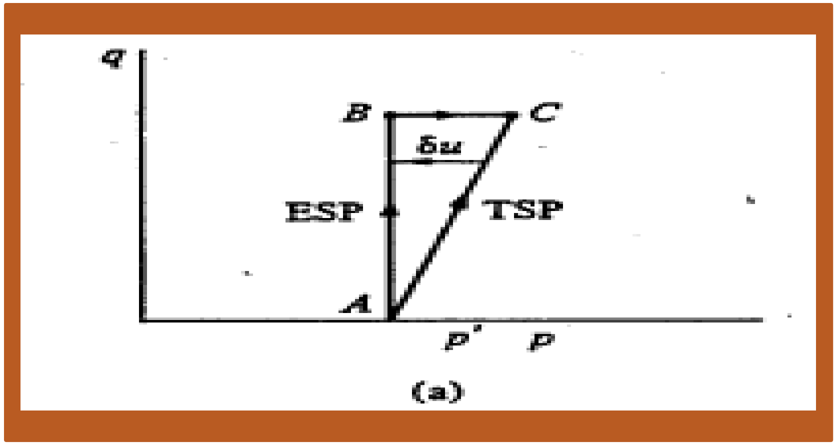
$$\delta\varepsilon_p = \frac{1}{K'}\delta p' \quad \Rightarrow \quad \frac{1}{K'}\delta p' = 0$$

There is no reason why the effective bulk modulus should be infinite , since the elastic properties of soil skeleton may not change :

$$\Rightarrow \quad \delta p' = 0 \quad \Rightarrow \quad \delta p = \delta u$$

Elasticity

- Therefore the effective stress path is vertical in p'-q plane:



Evidently , any total stress path may be imposed and still having the ESP vertical.

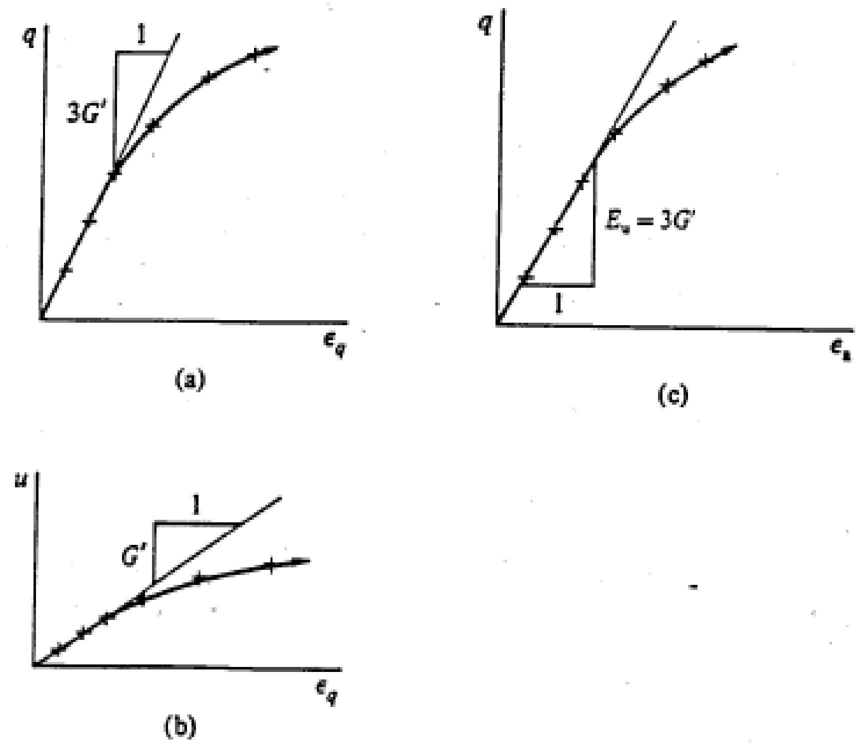
If the TSP is vertical then $\delta p = 0$ and since $\delta u = \delta p$

There is no P.W.P generated during the test.

Elasticity

- The behavior of the soil element is controlled by the change in effective stress , but in terms of total stress :

$$\begin{Bmatrix} \delta p \\ \delta q \end{Bmatrix} = \begin{bmatrix} K_u & 0 \\ 0 & 3G_u \end{bmatrix} \begin{Bmatrix} \delta \varepsilon_p \\ \delta \varepsilon_q \end{Bmatrix}$$



Elasticity

For undrained condition

$$\delta \varepsilon_p = 0 \quad \Rightarrow \quad \frac{\boldsymbol{\delta p}}{\boldsymbol{K_u}} = \mathbf{0}$$

There can be no constrained on the total stress path

$$\Rightarrow \quad K_u = \infty$$

$$\Rightarrow \quad \vartheta_u = 0.5$$

Elasticity

- Shear modulus is independent of the drinage condition .Since $G_u = G'$

$$\frac{E'}{2(1+\vartheta')} = \frac{E_u}{2(1+\vartheta_{u)}} \Rightarrow E_u = \frac{3E'}{2(1+\vartheta')}$$

Assignment # 2

E2.1

E2.2

E2.5

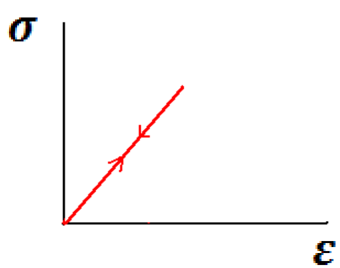
Chapter #3

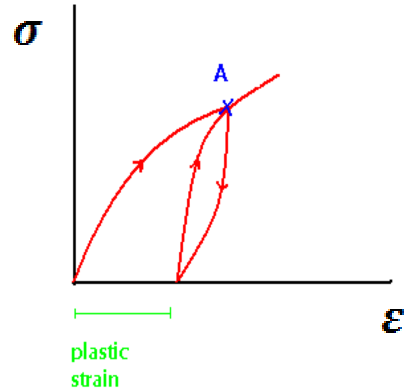
Plasticity & Yielding

Plasticity & Yielding

- In elasticity no energy is dissipated during application and removal of a load.

For many material and especially for soils , there is not a one-to-one relationship between stress and strain.





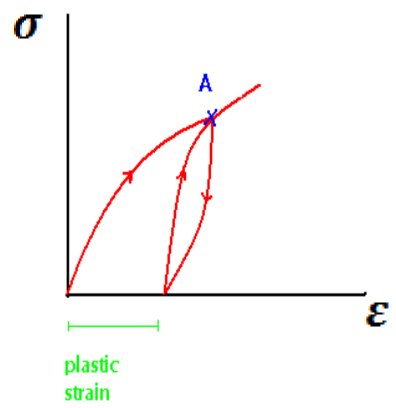
Plasticity & Yielding

Yield :

As the material departs from being elastic on reloading at point "A" (the past max load) , it is said to be "yielding".

The past max. load now becomes a "yield point".

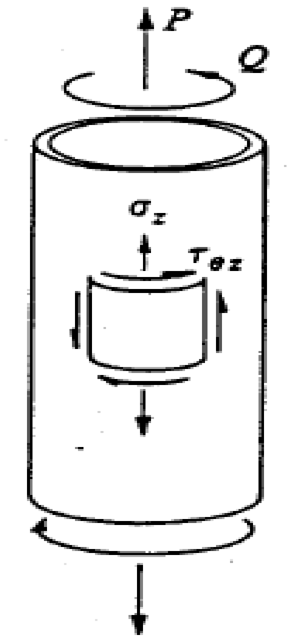
-Therefore, yielding is transition from a stiff response to a less stiff response.



Plasticity & Yielding

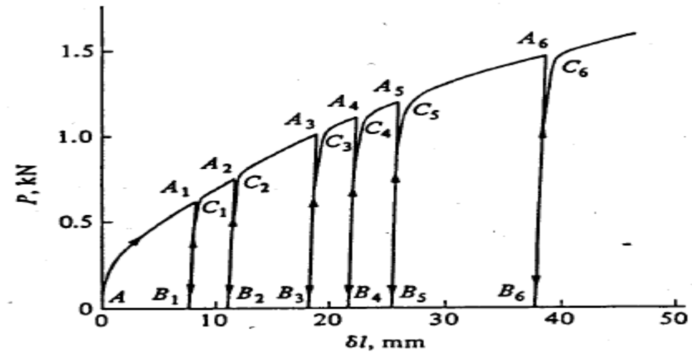
- Yielding of Metal tube in combined tension & torsion

"copper"



Plasticity & Yielding

a) Behavior in simple tension



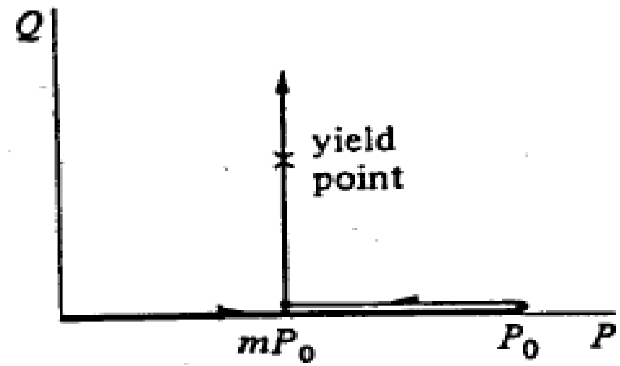
b) Behavior in simple torsion

-Similar behavior in pure tension & torsion.

Plasticity & Yielding

C) Behavior in combined tension & torsion :

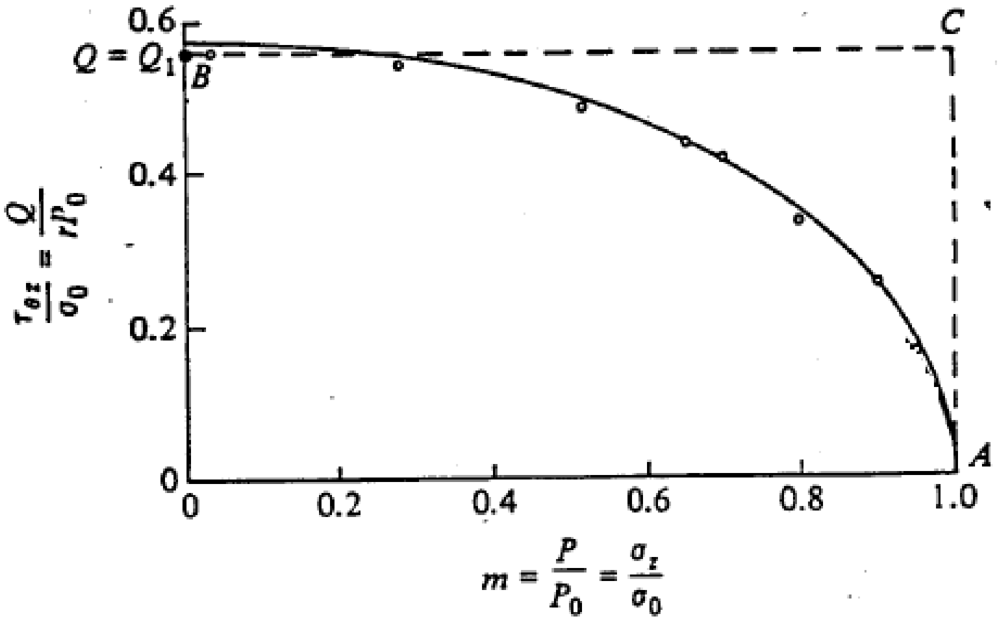
The stress path followed :



Perform the test with eight different values of "m" give eight different yield points.

Plasticity & Yielding

- A yield curve could be drawn defining the combination of tension and torsion for which yielding will occur as shown on the next figure.



Plasticity & Yielding

- NOTE :

1. If there was no interaction between the tension and torsion , the results would be on the rectangle.

2. This was a yield curve for the specimen with the particular history of preloading.

   This shape of yield curve may be predicted using a proper "yield criterion".

Plasticity & Yielding

- Yield Criteria

1- Tresca yield Criterion

Yielding occurs when the max shear stress reaches a critical value.

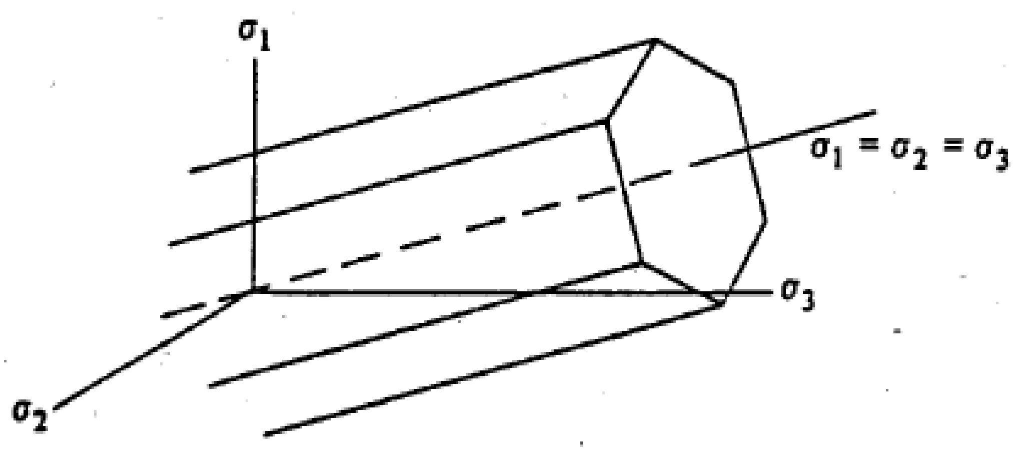
In terms of principal stresses :

$$\max(\sigma_i - \sigma_j) = 2c \qquad (i, j = 1, 2, 3)$$

2c : yield stress in uniaxial tension (σ_y)

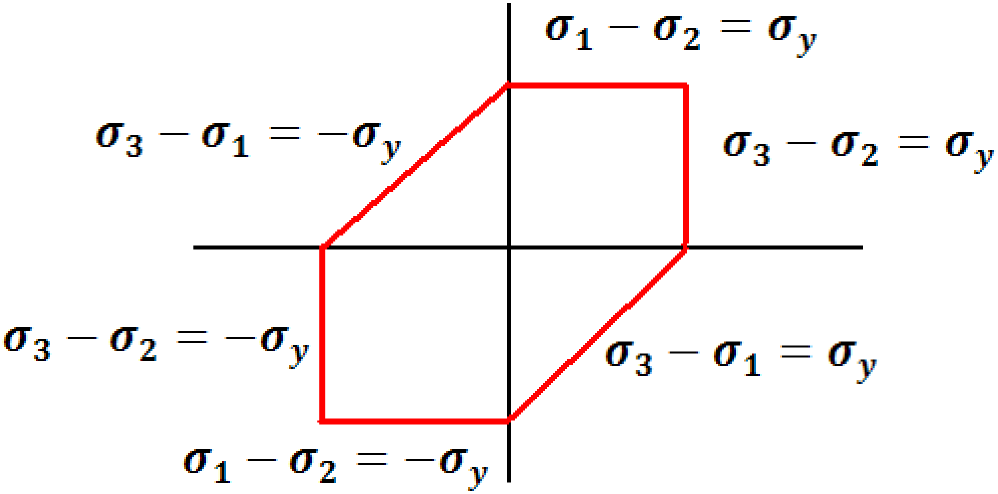
Plasticity & Yielding

- The yield criteria may be shown on a spaced formed by three axes σ_1 , σ_2 , σ_3 known as "Principal Stress Space".



Plasticity & Yielding

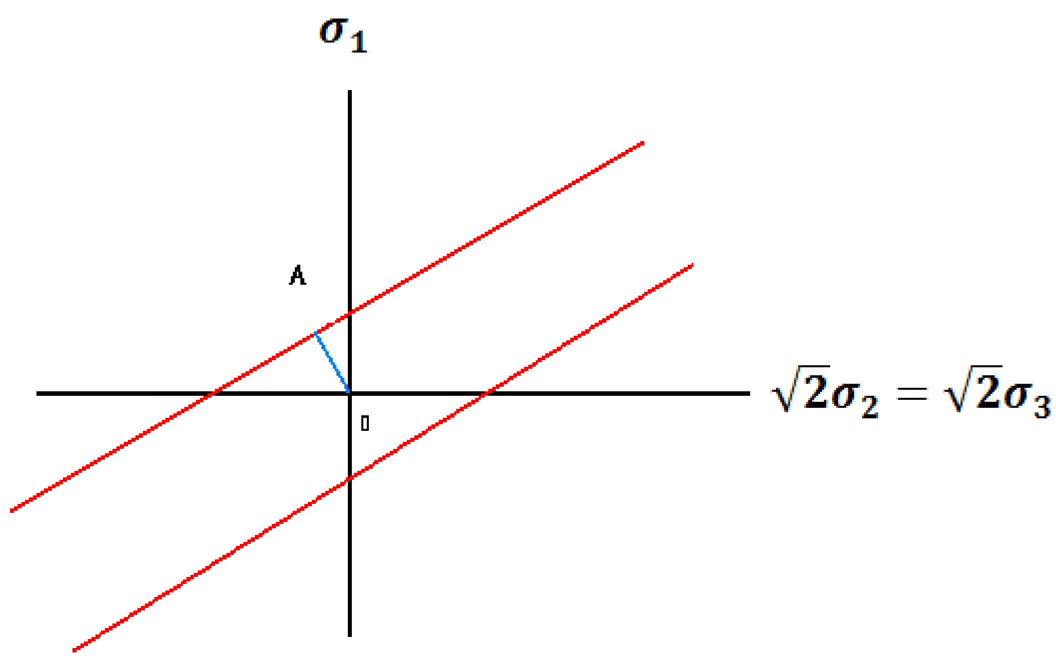
- Plane stress : ($\sigma_2 = 0$)



Plasticity & Yielding

- Triaxial : ($\sigma_2 = \sigma_3$)

$$\overline{OA} = \sqrt{\frac{2}{3}}\sigma_y$$



Plasticity & Yielding

- In $\tau - \sigma$ plane

Mohr circle

$$\sigma_1 , \sigma_3 = \frac{\sigma_z}{2} \pm \sqrt{(\frac{\sigma_z}{2})^2 + \tau_{\theta z}^2}$$

$$\frac{\sigma_1 - \sigma_3}{2} = \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}$$

$$\sigma_1 - \sigma_3 = 2c$$

$$\therefore \quad (\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2 = c^2$$

$$(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2 = 4c^2$$

Plasticity & Yielding

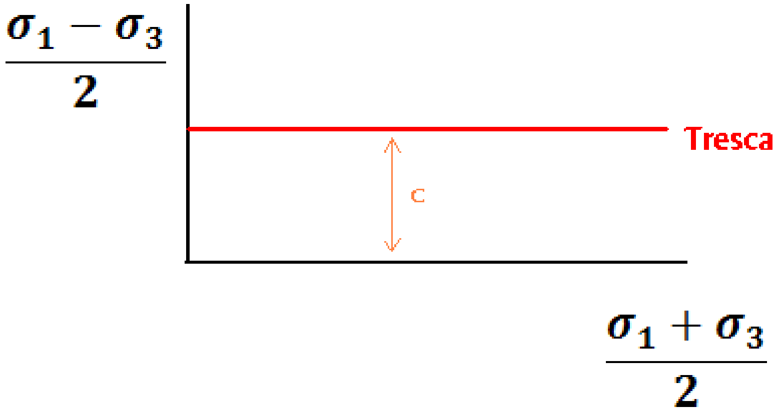
- For the previous example : ($\sigma_x = 0$)

$$\therefore \quad \sigma_y^2 + 4\tau_{xy}^2 = 4c^2$$

which is an ellipse

in pure torsion

$$\tau_{xy} = c = \frac{1}{2}\sigma_y$$



$$2c = \sigma_y$$

$$c = \frac{\sigma_y}{2}$$

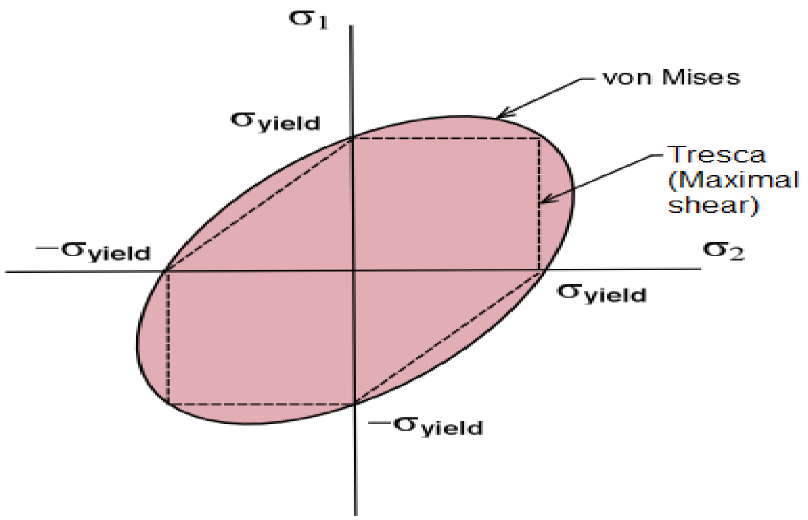
Plasticity & Yielding

2- Von Mises Yield Criteria

Yielding occurs when the second invariant of the stress tensor reaches a critical value.

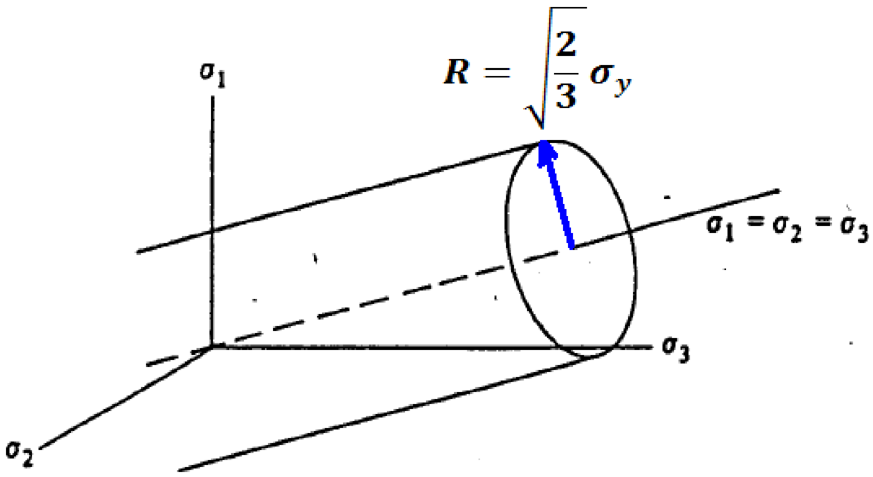
$$(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2+(\sigma_1-\sigma_2)^2=8c^2$$

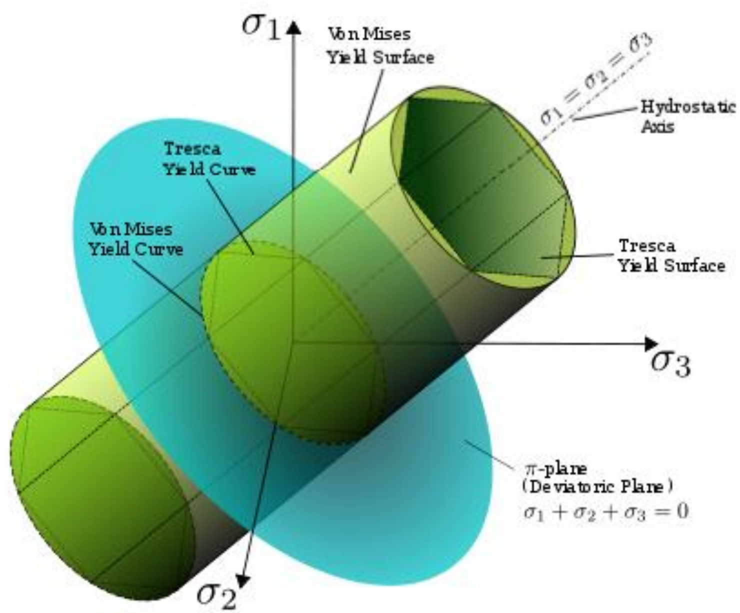
- Plane stress $(\sigma_2 = 0)$



Plasticity & Yielding

- Principal stress-space





Plasticity & Yielding

- For previous Example :

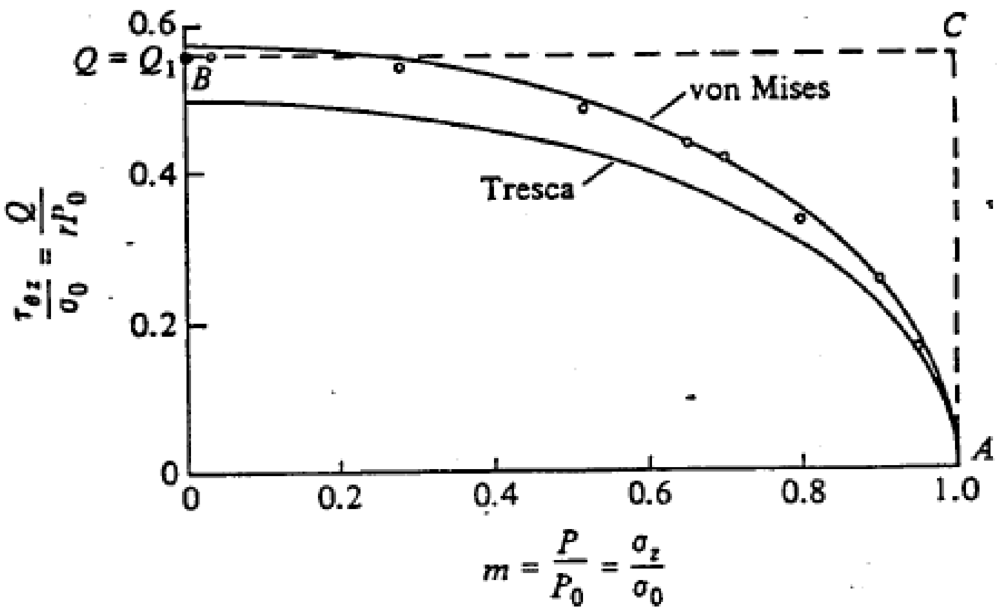
$$\sigma_z^2 + 3\tau_{\theta z}^2 = 4c^2$$

An ellipse again in pure torsion : ($\sigma_z = 0$)

$$\tau_{\theta z} = \frac{2c}{\sqrt{3}} = \frac{\sigma_y}{\sqrt{3}}$$

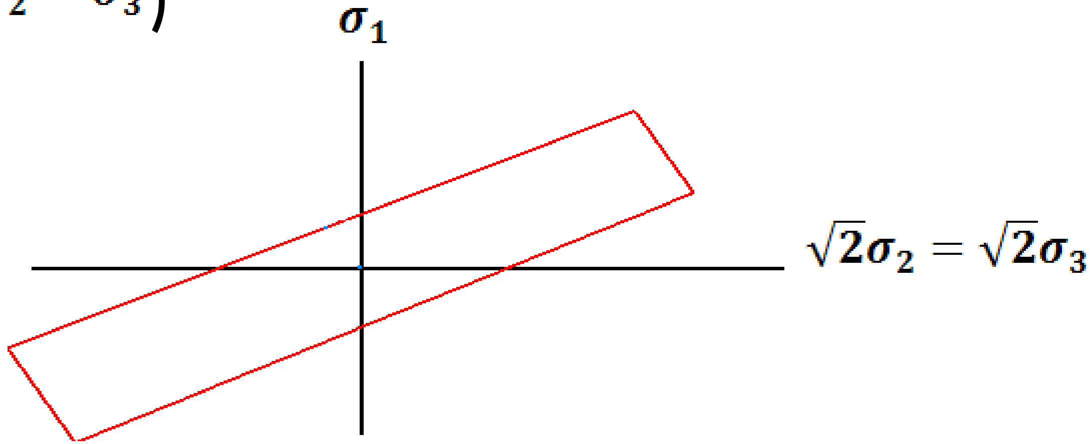
Plasticity & Yielding

- This figure indicates that Von Mises criterion describes the yield surface for anealed copper better than Tresca criterion.

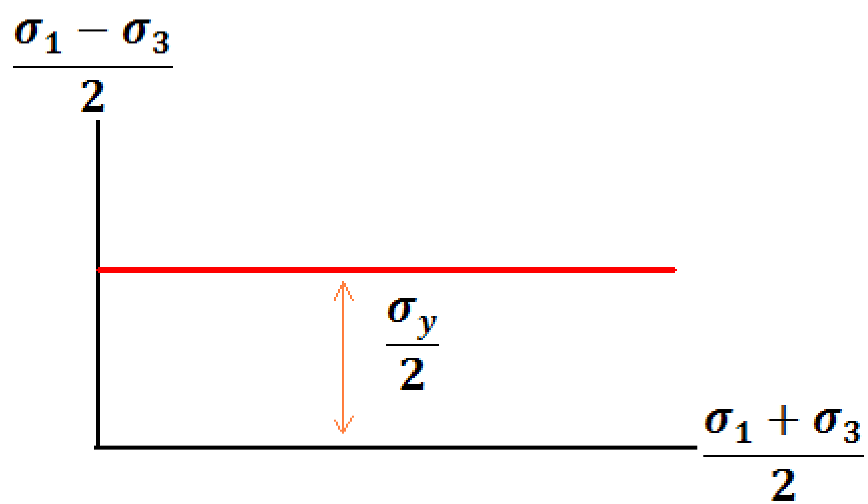


Plasticity & Yielding

- Triaxial ($\sigma_2 = \sigma_3$)



- In $\tau - \sigma$ plane :



$$2(\sigma_1 - \sigma_3)^2 = 2\sigma_y^2$$

$$\sigma_1 - \sigma_3 = \sigma_y$$

Plasticity & Yielding

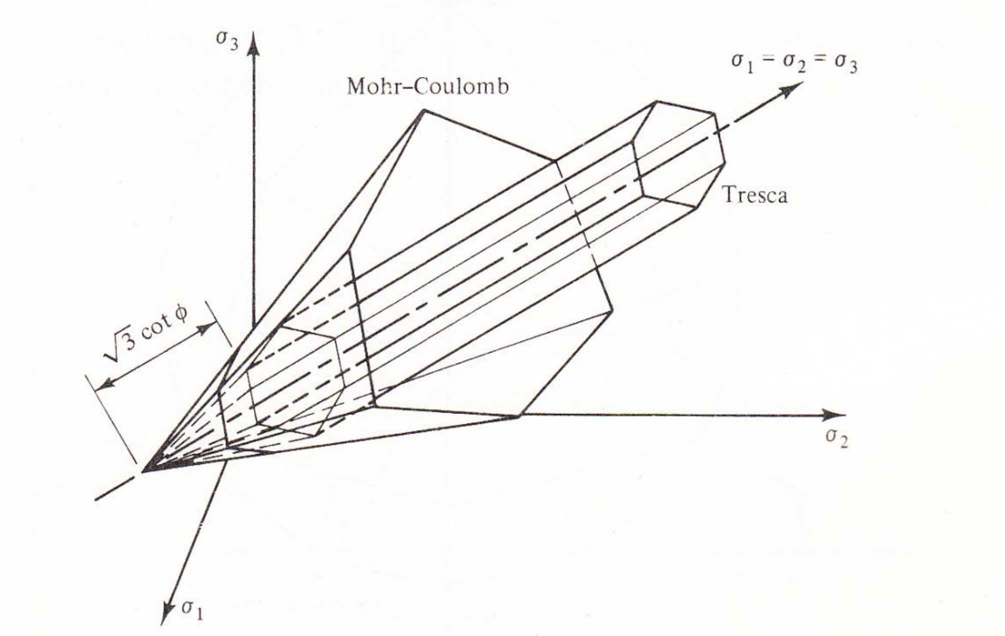
3- Mohr-Coulomb Yield Criteria

$$\frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_1 + \sigma_3}{2} \sin\varphi + c\cos\varphi$$

-In general :

$$\sigma_{max} = \sigma_{min} N^2 + 2cN$$

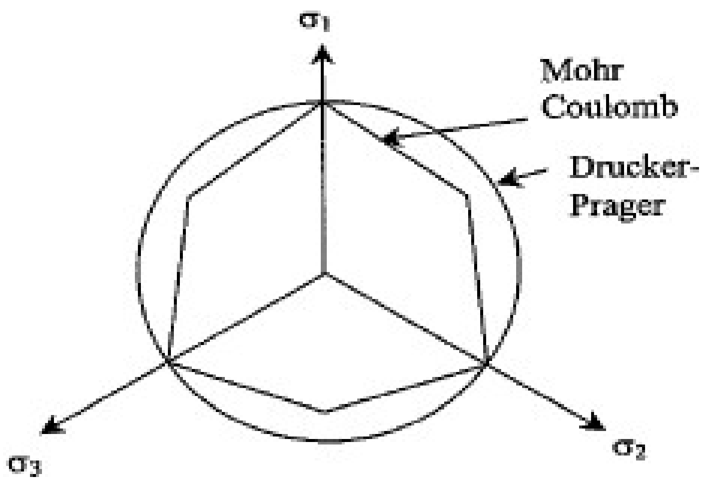
$$N = \tan(45 + \frac{\varphi}{2})$$



Plasticity & Yielding

$\pi - plane$: Plane perpendicular to space diagonal

The failure or yield criteria on $\pi - plane$:



Plasticity & Yielding

For the previous example using "Mohr-Coulomb" criterion :

$$\frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_1 + \sigma_3}{2} sin\varphi + c\, cos\varphi$$

$$\sigma_1 = \frac{\sigma_z}{2} + \sqrt{(\frac{\sigma_z}{2})^2 + \tau_{\theta z}^2}$$

$$\sigma_3 = \frac{\sigma_z}{2} - \sqrt{(\frac{\sigma_z}{2})^2 + \tau_{\theta z}^2}$$

$$\frac{\sigma_1 + \sigma_3}{2} = \frac{\sigma_z}{2}$$

Plasticity & Yielding

$$\sqrt{(\frac{\sigma_z}{2})^2 + \tau_{\theta z}^2} = \frac{\sigma_z}{2} sin\varphi + c\, cos\varphi$$

$$(\frac{\sigma_z}{2})^2 + {\tau_{\theta z}}^2 = (\frac{\sigma_z}{2})^2 sin^2\varphi + c^2\, cos^2\varphi + c\, cos\varphi\, \sigma_z\, sin\varphi$$

$$(\frac{\sigma_z}{2})^2[1 - sin^2\varphi] + \tau_{\theta z}^2 = c^2\, cos^2\varphi + c\, cos\varphi\, sin\varphi\, \sigma_z$$

$$(\frac{\sigma_z}{2})^2\, cos^2\varphi - c\, cos\varphi\, sin\varphi\, \sigma_z + \tau_{\theta z}^2 = c^2 cos^2\varphi$$

$$(\frac{\sigma_z}{2} cos\varphi - c\, sin\varphi)^2 - c^2 sin^2\varphi + \tau_{\theta z}^2 = c^2 cos^2\varphi$$

$$(\frac{\sigma_z}{2}\, cos\varphi - c\, sin\varphi)^2 + \tau_{\theta z}^2 = c^2$$

ELLIPSE

Plasticity & Yielding

For pure torsion ($\sigma_z = 0$)

$$\Rightarrow \quad \tau_{\theta z} = c\cos\varphi$$

$$if \;\; \varphi = 0 \;\; \rightarrow \;\; \sigma_y = +2c \;, \;\; \tau_{\theta z} = c = \frac{1}{2}\sigma_y$$

$$otherwise \;\; if \;\; \varphi \neq 0 \;\; \rightarrow \;\; \tau_{\theta z} < c$$

Plasticity & Yielding

The result of the previous example on $\pi - plane$:

$$\frac{\tau_{\theta z}}{\sigma_0} = 0.4 \qquad \frac{\sigma_z}{\sigma_0} = 0.7$$

$$\begin{matrix}\sigma_1 \\ \sigma_3\end{matrix} = \frac{\sigma_z}{2} \pm \sqrt{(\frac{\sigma_z}{2})^2 + \tau_{\theta z}^2} = \frac{0.7\sigma_0}{2} \pm \sqrt{(0.35\sigma_0)^2 + (0.4\sigma_0)^2}$$

$$= (o.35 \pm 0.53)\sigma_0 \quad \rightarrow \quad \begin{cases} \sigma_1 = 0.88\ \sigma_0 \\ \sigma_3 = -0.18\ \sigma_0 \end{cases}$$

$$2c = \sigma_0$$

Plasticity & Yielding

4- Drucker Prager Model

A generalization of Mohr-Coulomb to account for all principal stresses :

$$f = \sqrt{J_{2D}} - \alpha J_1 - K$$

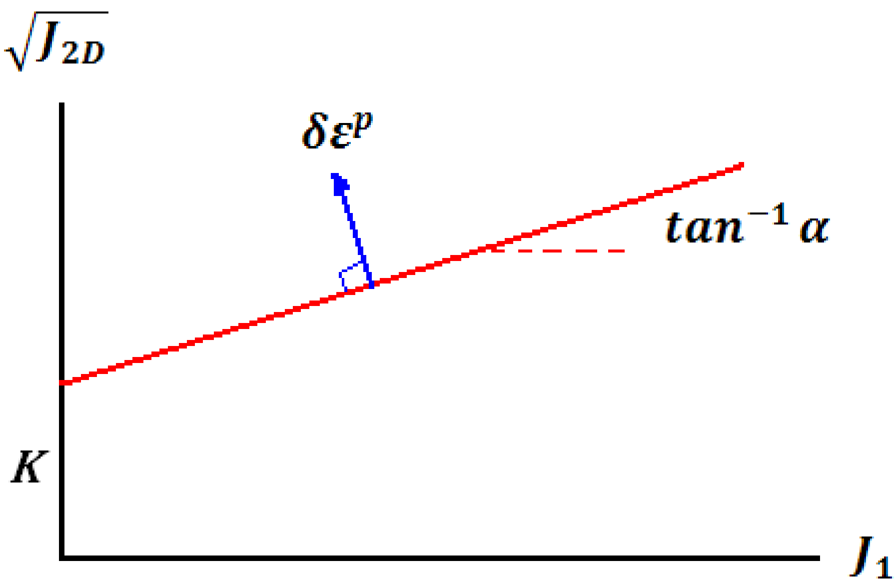
J_1 : first invariant of stress tensor

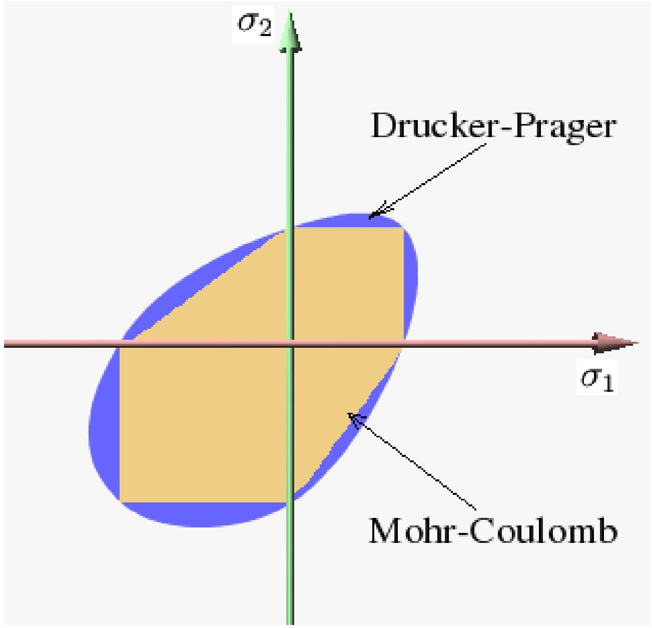
J_{2D} : 2nd invariant of deviatoric stress tensor

α & K : positive material parameters

Plasticity & Yielding

- Drucker prager criteria





$$S_{ij} = Deviator\ stress\ tensor = \sigma_{ij} - \frac{J_1}{3}\delta_{ij}$$

$$S_{ii} = 0$$

Plasticity & Yielding

- For c & φ from triaxial test condition :

$$\alpha = {2 \sin\varphi}\Big/{\sqrt{3}(3-\sin\varphi)}$$

$$K = {6c \cos\varphi}\Big/{\sqrt{3}(3-\sin\varphi)}$$

$$\delta\varepsilon_{ij}^{p} = \aleph \frac{\partial f}{\sigma_{ij}} = \aleph \left[\frac{S_{ij}}{2\sqrt{J_{2D}}} - \alpha\delta_{ij} \right]$$

$$\therefore \ \delta\varepsilon_{ii}^{p} = -3\aleph\alpha \qquad (always \ \ negative)$$

Incremental plastic volumetric strain

Plasticity & Yielding

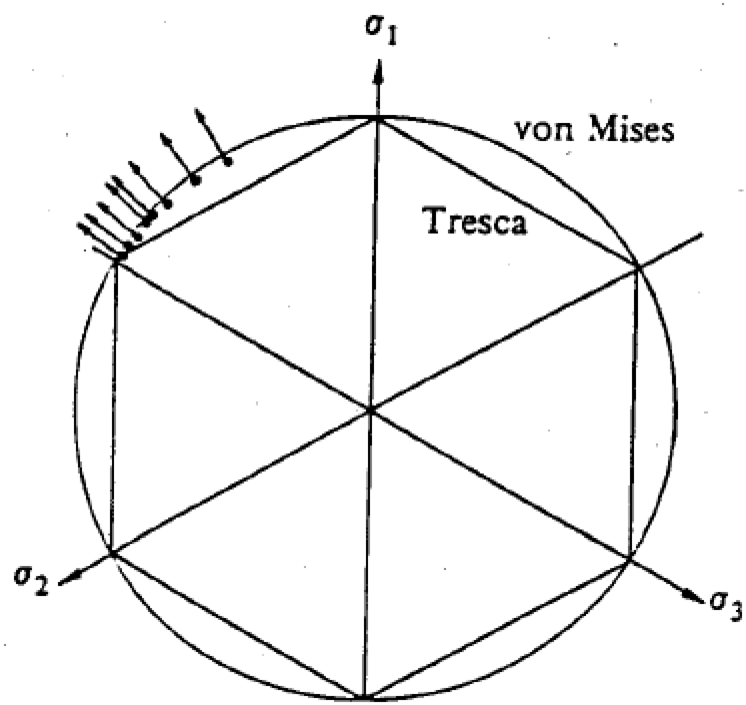
NORMALITY :

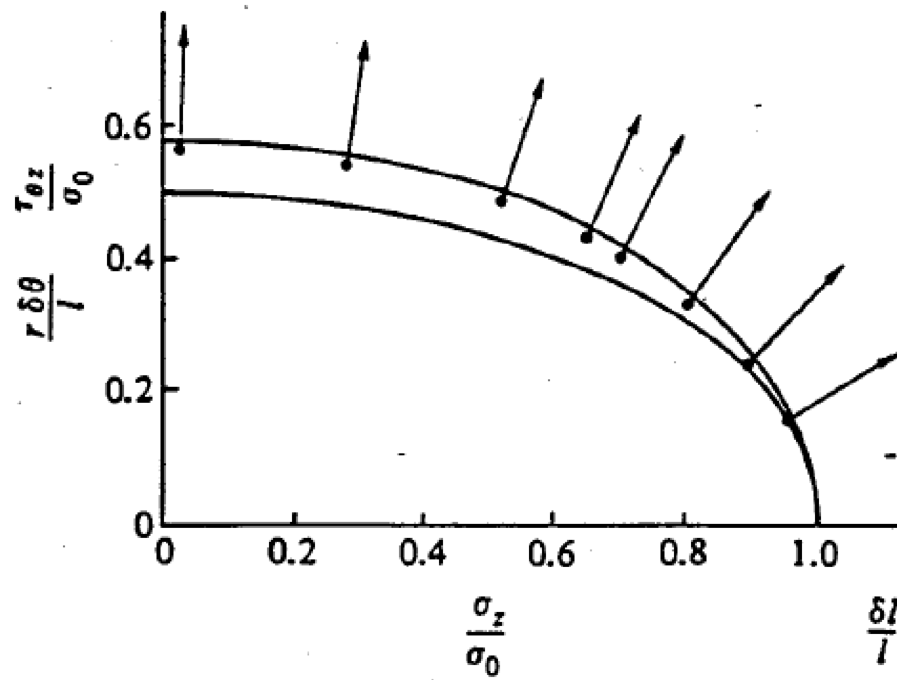
In principal stress space the correct strain parameters to associate with the principal stresses are the principal strain increments.

For isotropic material the principal axes of stress and strain increments are coincident after yield.

The plastic strain increment vectors are plotted in next slide :

Plasticity & Yielding





Plasticity & Yielding

The direction of each vector indicates the relative amounts of plastic twist and extension that occur when yield is reached.

These vectors were approximately normal to the Von Mises criterion.

From these figures :

-Direction of plastic strain increment vectors Do not depend on the route followed to the specific point on the yield curve but "depend on the particular combination of the stresses at the particular point at which yielding has occurred".

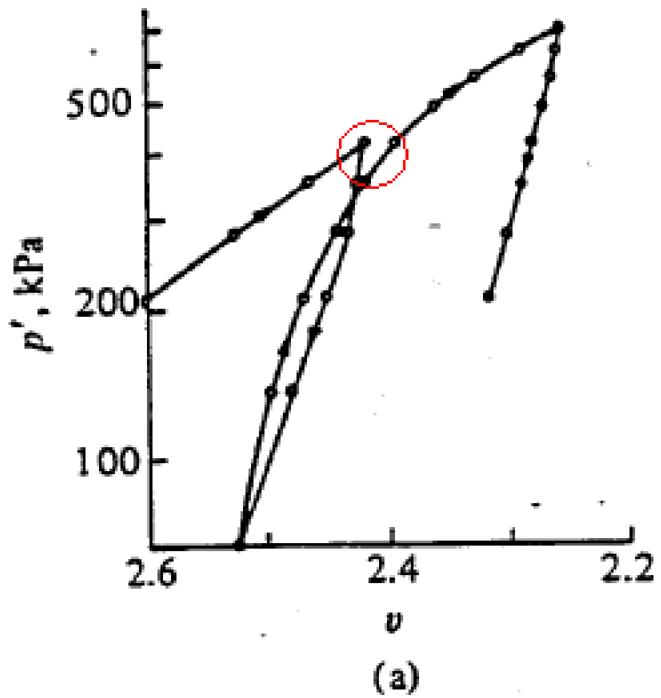
The mechanism of plastic deformation depends on stresses and not stress increments.

Plasticity & Yielding

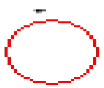
Yielding of Clays

Yielding occurs following different stress path :

1-Isotropic compression

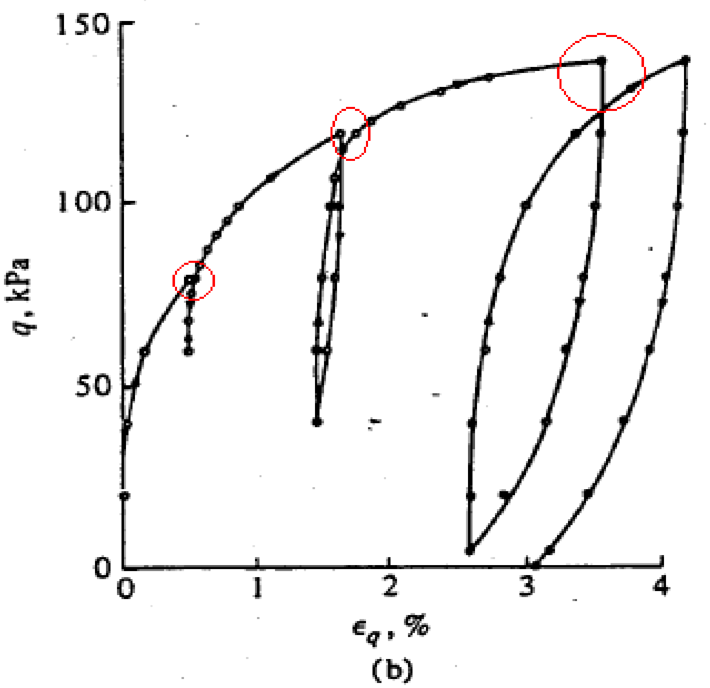


ν : specific volume=1+e



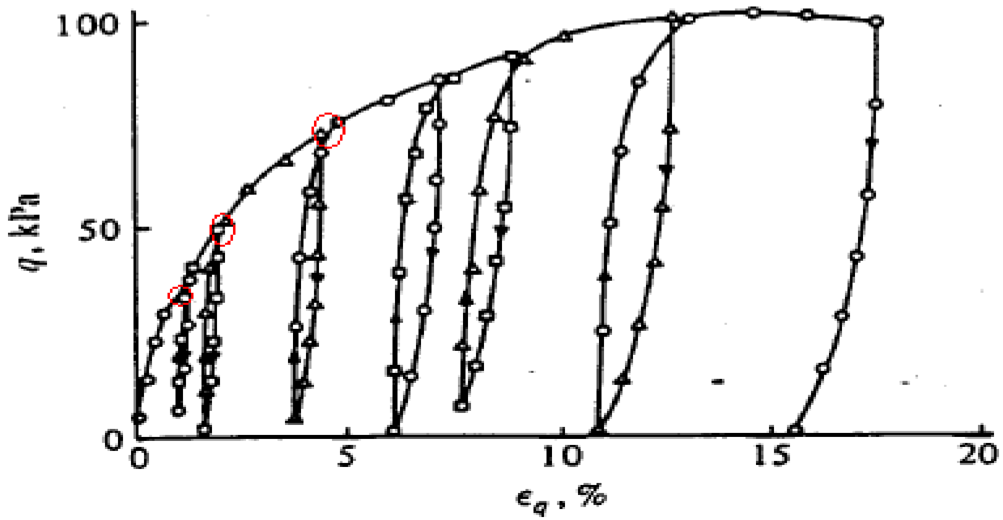
Plasticity & Yielding

2-Undrained triaxial compression



Plasticity & Yielding

3-Compression and unloading at constant P’



(c)

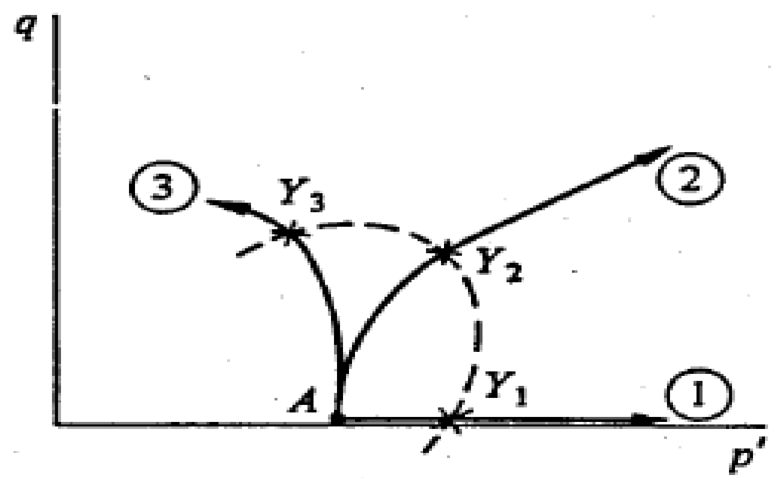
Plasticity & Yielding

Results :

-Preconsolidation pressure is a yield point.

-For stresses < σ'_{vc} → Elastic behavior

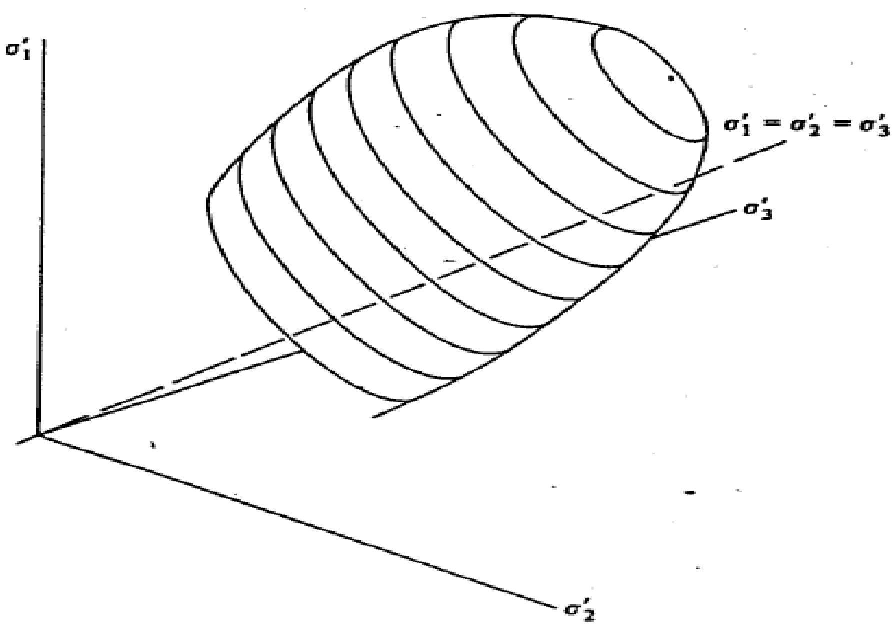
-The result of yielding on three different stress paths for a soil on p-q diagram :



Plasticity & Yielding

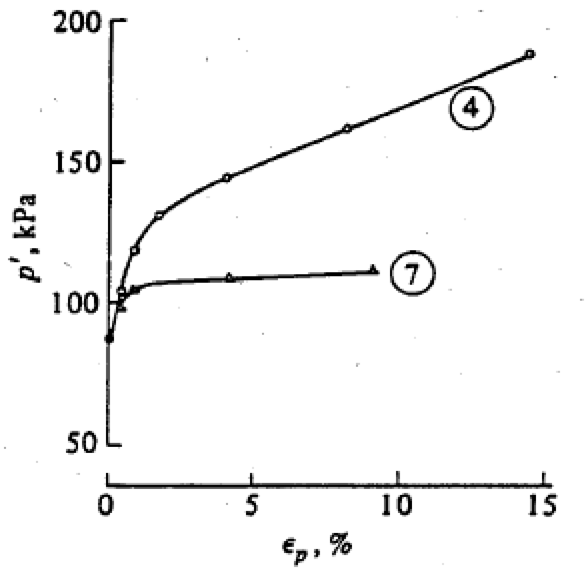
The yielding surface may be regarded as a generalized preconsolidation pressure for different stress paths.

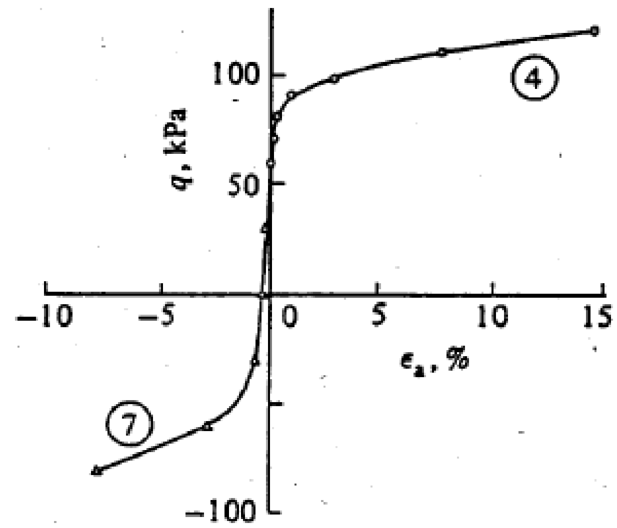
This yield curve may be sketched in principal stress space :



Plasticity & Yielding

How to determine the yield point from experimental data ?



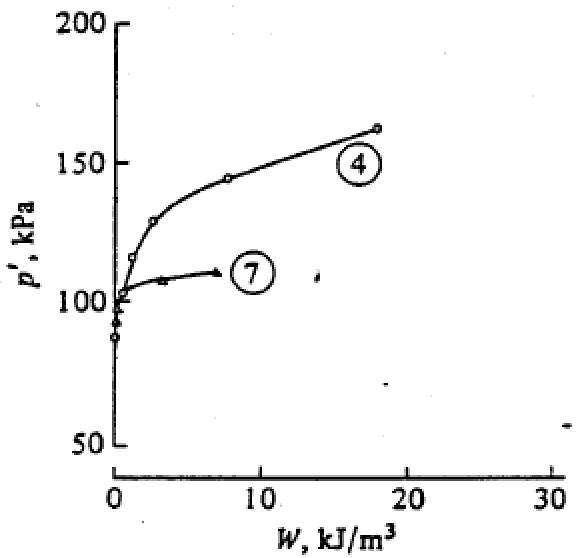


4 & 7 : different stress paths

Plasticity & Yielding

For triaxial apparatus :

$$W = \int (p' \, d\varepsilon_p + q \, d\varepsilon_q)$$



$$W = \int \sigma_a \, d\varepsilon_a$$

Plasticity & Yielding

Use as many plots as possible , then "average"

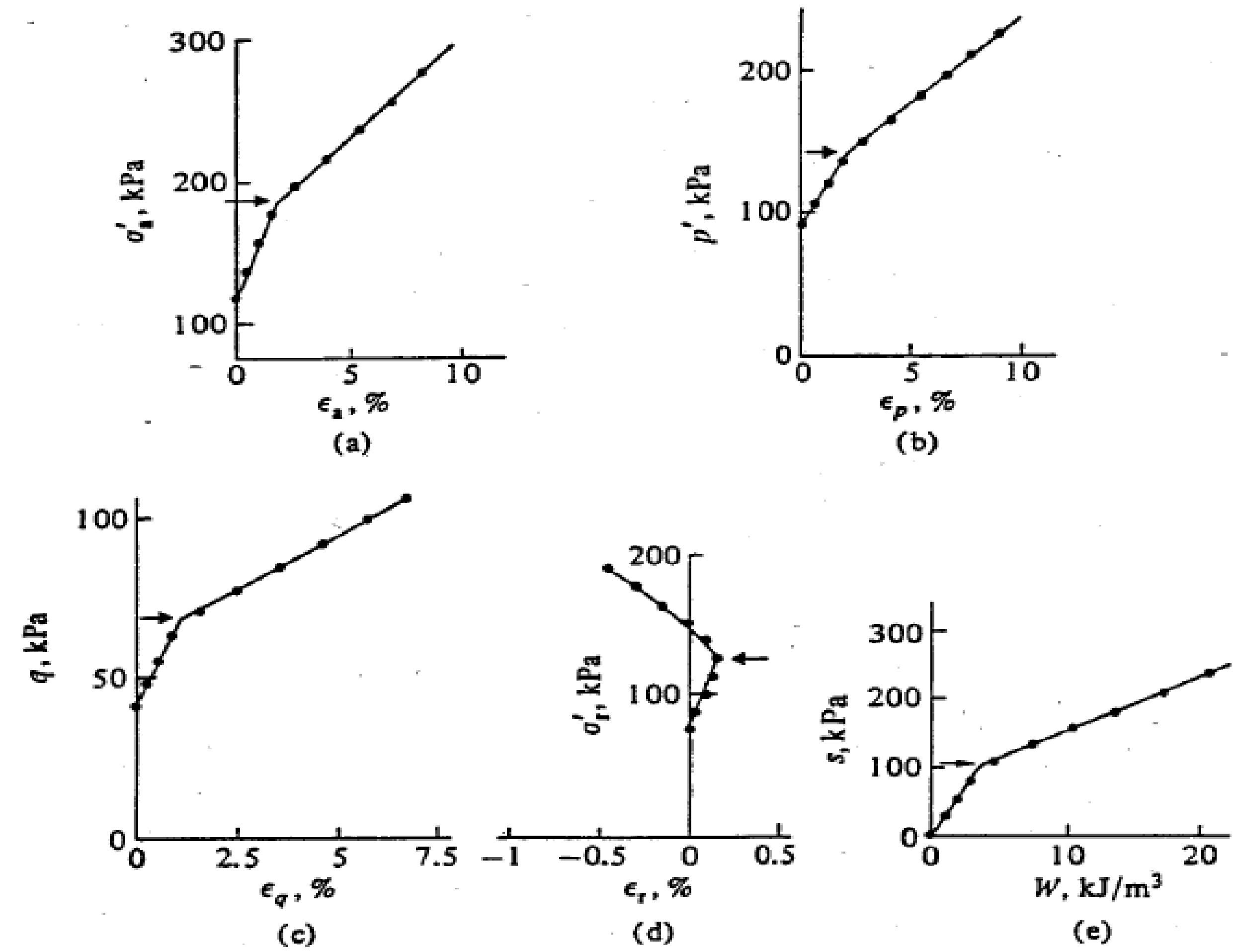
Also "S" versus "W" where

LENGTH OF STRESS PATH

$$\delta s = \sqrt{\delta p'^2 + \delta q^2}$$

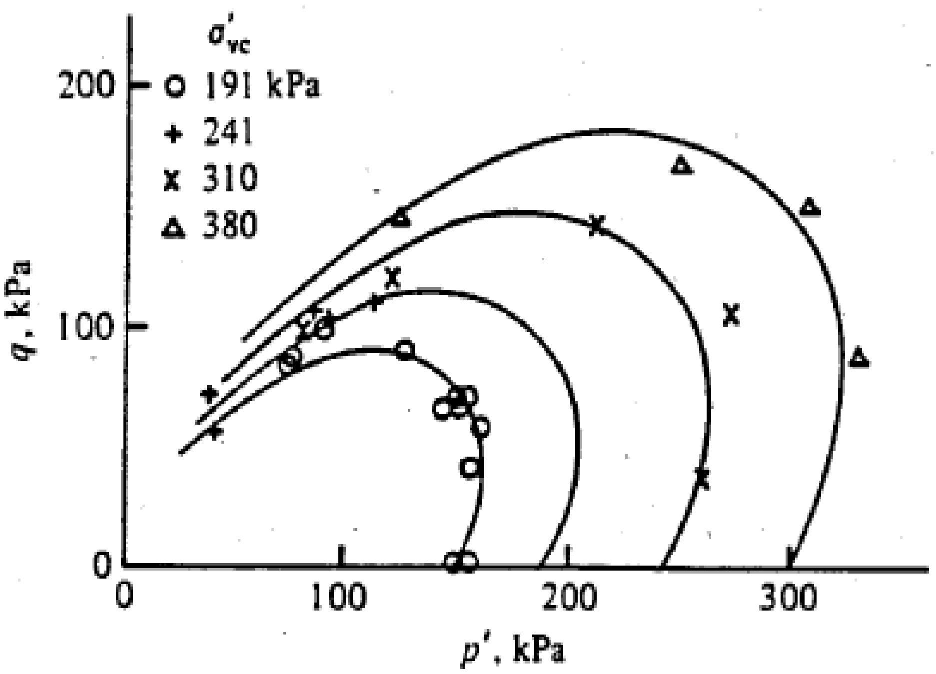
Plasticity & Yielding

For example :



Plasticity & Yielding

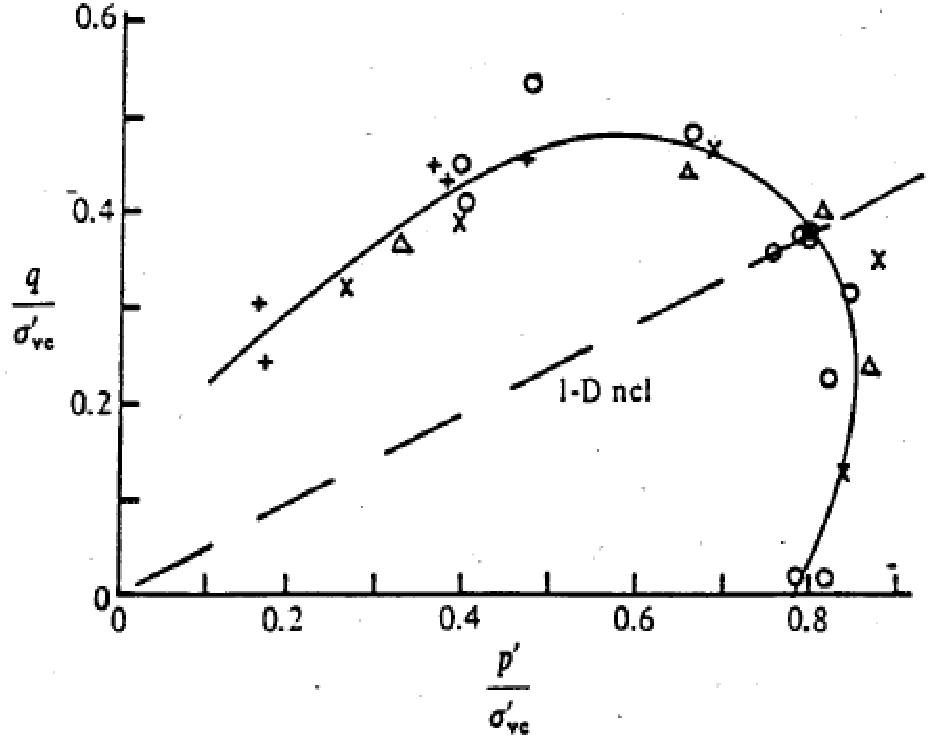
Yield curves for soil of different preconsolidation pressure :



(a)

Plasticity & Yielding

If Normalized :



(b)

Plasticity & Yielding

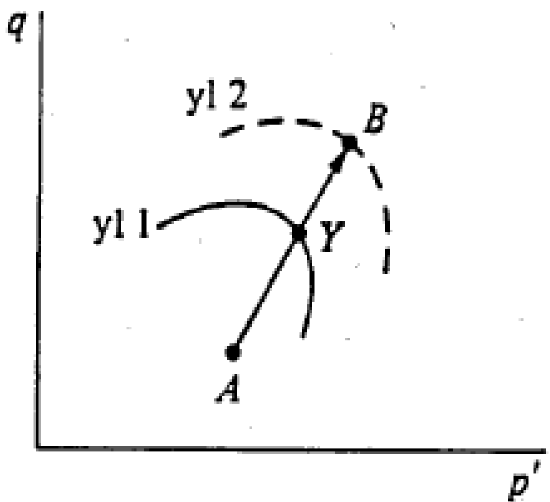
Yielding of SANDs :

The determination of the entire shape of a current yield surface for a given soil sample is not feasible :

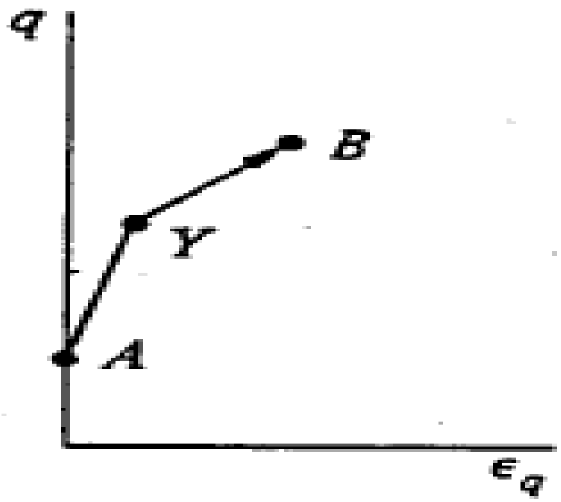
a) Having identical samples (sample of undisturbed sand) is nearly impossible.
b) To investigate current yield point , it is needed to go beyond the current yield point.

Plasticity & Yielding

Y: current yield → B: subsequent yield point



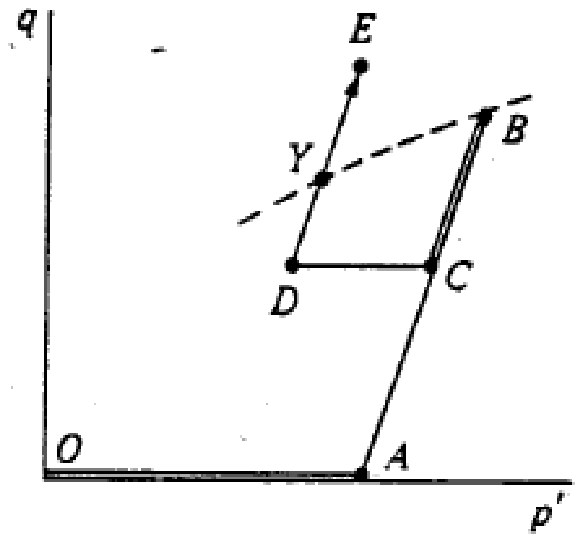
(a)

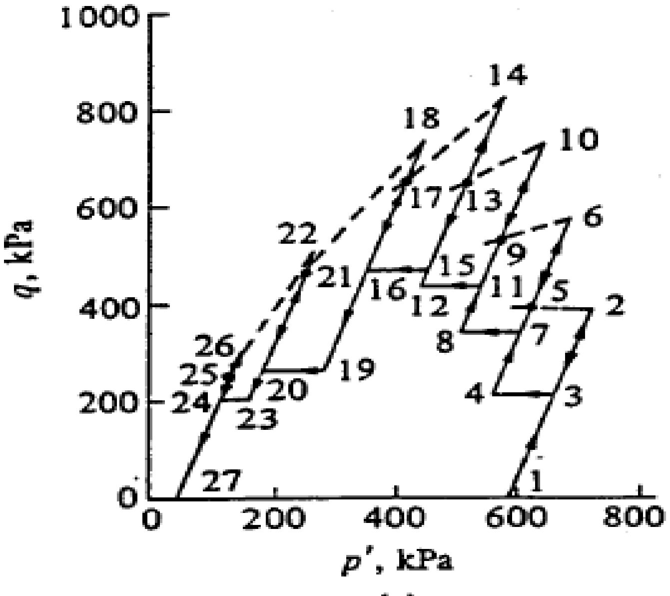


(b)

Plasticity & Yielding

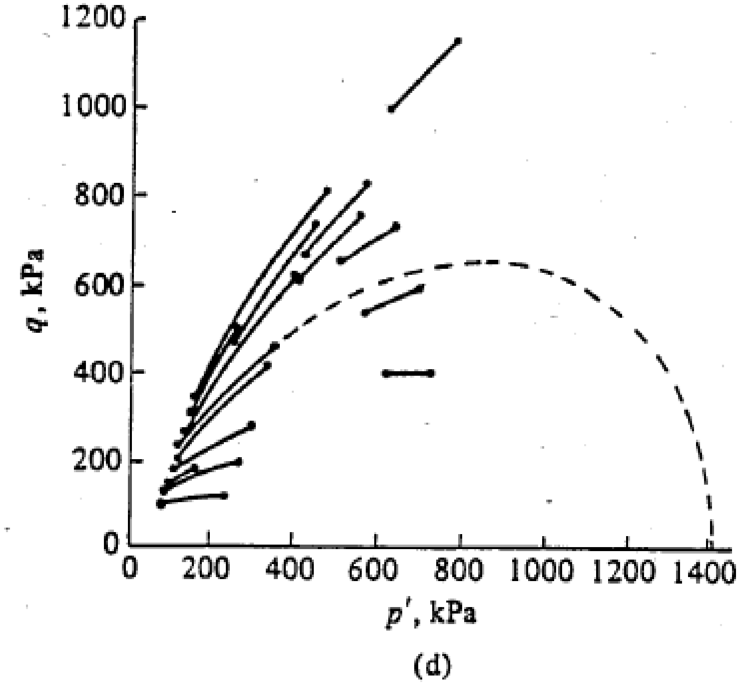
Path followed in triaxial test :





(a)

Plasticity & Yielding



Plasticity & Yielding

HW :

E3.1c - E3.5

Chapter # 4

Elastic-Plastic Model For Soils

Elastic-Plastic Model

Introduction :

Yield surface :

-stress changes inside yield surface (Elastic response).

-stress changes engaging the yield surface (Elastic and plastic deformation).

Elastic-Plastic Model

-Magnitude of plastic deformation?

-Relative magnitude of components?

-Link between these magnitudes and yield surface?

p ε_p

q ε_q

-Changing size of the yield surface

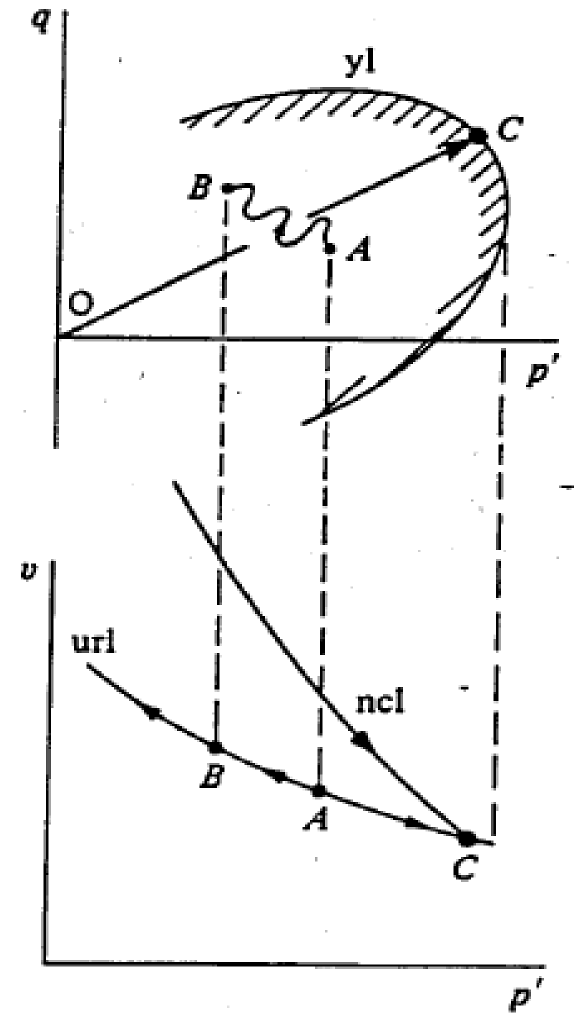
-Related to changes in volume → " Volumetric Hardening"

Elastic-Plastic Model

Elastic behavior :

Isotropic , Elastic :

$$\begin{bmatrix} \delta\varepsilon_p \\ \delta\varepsilon_q \end{bmatrix} = \begin{bmatrix} 1/K' & 0 \\ 0 & 1/3G' \end{bmatrix} \begin{bmatrix} \delta p' \\ \delta q \end{bmatrix}$$



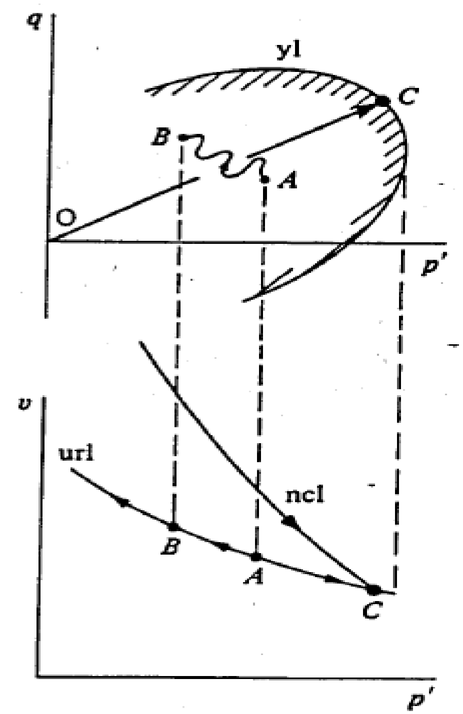
Elastic-Plastic Model

"A" to "B" ➤ change in volume (δv)

This volume change does not depend on the stress path followed from "A" to "B".

Stress path independent because the Response is elastic and

$$\delta \varepsilon_p = \frac{1}{K'} \delta p'$$



Elastic-Plastic Model

"OC" ❯ ncl on the volume change plane

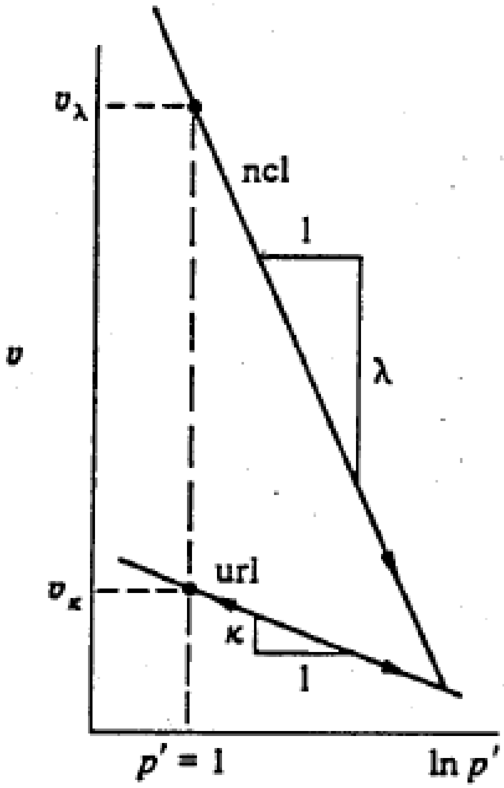
During the normal compression, irrecoverable deformations occur while pushing the yield surface to its present position.

Elastic-Plastic Model

The compression and unloading curve becomes linear on a $v - lnp'$ plot :

$$NCL: \quad v = v_\lambda - \lambda \, lnp' \qquad (1)$$

$$URL: \quad v = v_\kappa - \kappa \, lnp' \qquad (2)$$



v_κ & v_λ : depend on the units chosen for p'

Elastic-Plastic Model

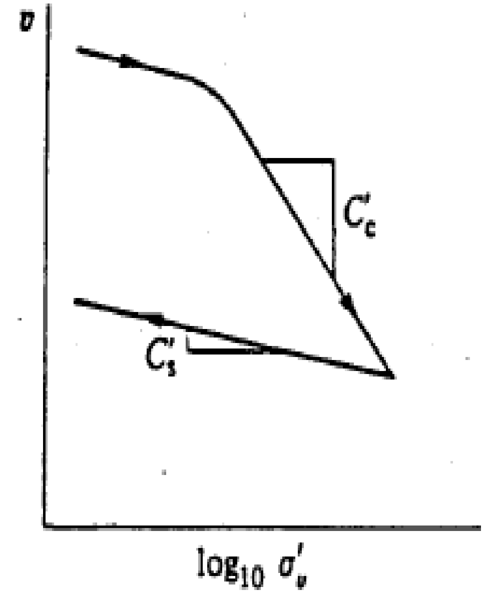
$$C'_c = \lambda \ln 10 \simeq 2.3\lambda$$

$$C'_s \simeq \kappa \ln 10 \simeq 2.3\kappa$$

C'_c

C'_s

From conventional plot
of oedometer test result



Elastic-Plastic Model

$$from\,(2) \quad : \quad \delta v^e = -k\frac{\delta p'}{p'}$$

$$knowing \quad \delta\varepsilon_p^e = \frac{-\delta v^e}{v} \;\Rightarrow\; \delta\varepsilon_p^e = k\frac{\delta p'}{vp'} \quad (3)$$

$$\delta\varepsilon_p^e = \frac{1}{k'}\delta p' \;\Rightarrow\; K' = \frac{vp'}{k} \quad (4)$$

Elastic-Plastic Model

Note :

Eq.(4) ➔ Bulk modulus increases with mean pressure p'

The effect of a small reduction in "v" is negligible compared to the increase in p'.

-Within the yield surface δq ➔ no volume change

Elastic-Plastic Model

$$\delta\varepsilon_q^e = \frac{1}{3G'}\delta q$$

$$G' = \frac{3(1-2v')K'}{2(1+v')} \qquad (5)$$

G' increases with mean pressure

Elastic-Plastic Model

-A constant value of shear modulus might be assumed

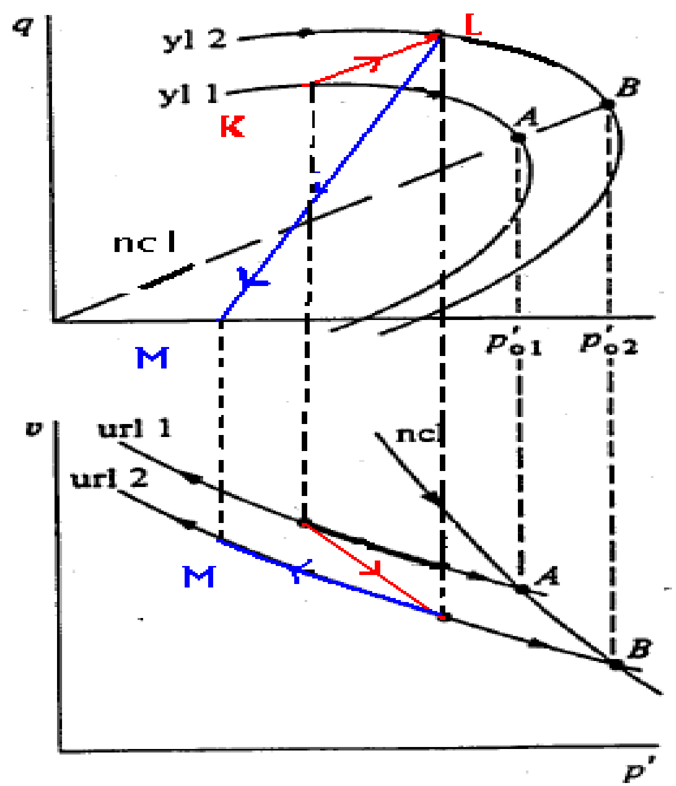
$$v' = \frac{3K' - 2G'}{2G' + 6K'} \qquad (6)$$

V' varies with mean pressure

Elastic-Plastic Model

Plastic Volumetric Hardening

Consider a change in stress that causes the soil to yield :



Elastic-Plastic Model

"K" to "L" → "L" on new yield lucas

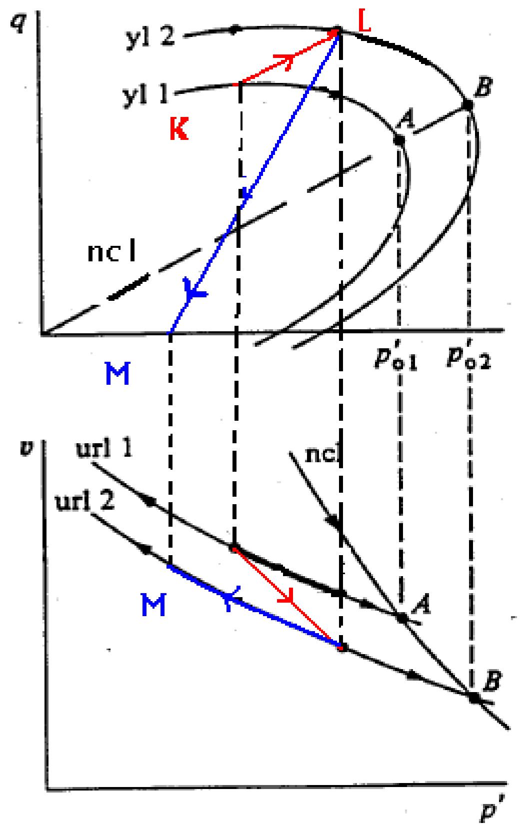
Assuming that the size of yield curve depends on the preconsolidation pressure as was observed before for Winnipeg clay.

→ "Shape is the same , Size changing with σ'_{vc}

Elastic-Plastic Model

y2 has the same shape as y1

- y1 corresponds to a normal consolidation to point "A"
  - point "K" on unloading
- y2 corresponds to a normal consolidation to point "B"
  - point "L" on unloading



Elastic-Plastic Model

Note :

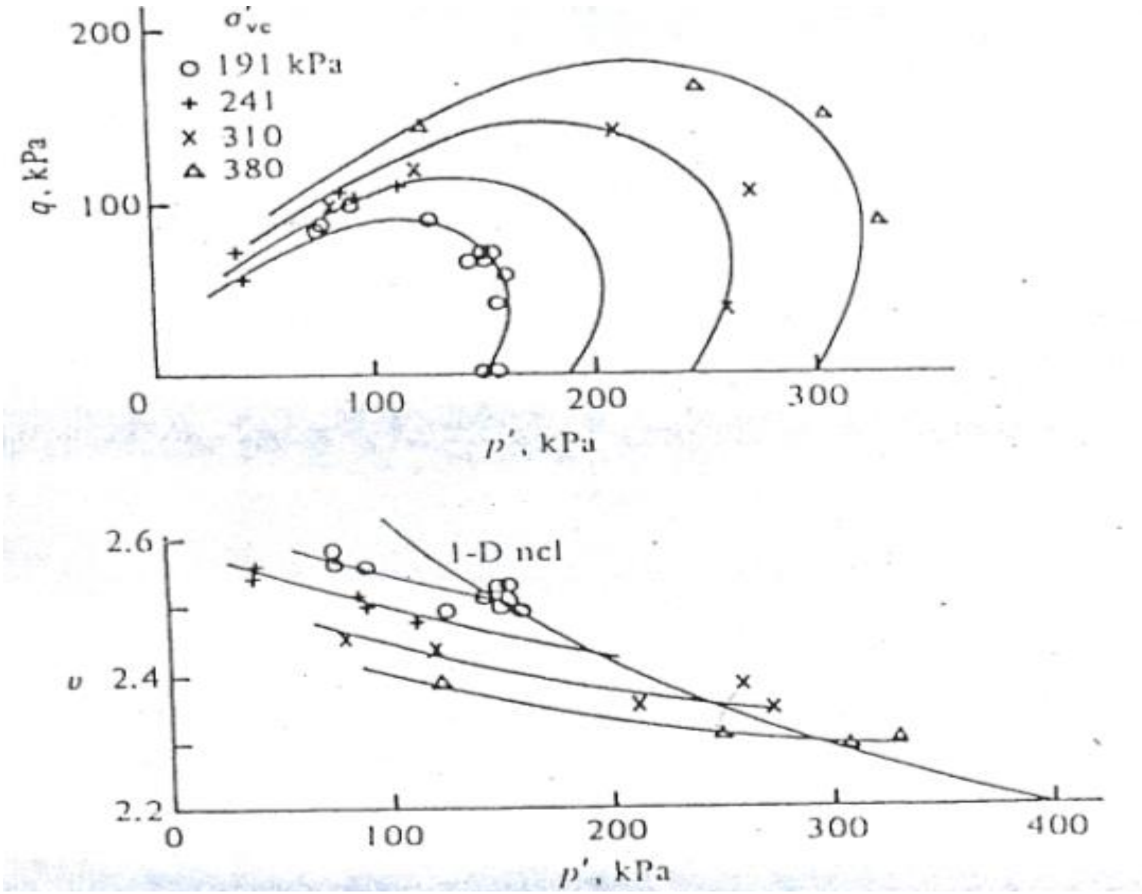
When a new yield surface y2 is reached for the soil , its past history can not be investigated.

If for example the soil is unloaded from point "L" to isotropic condition at point "M", a researcher may find the current yield surface but can not deduce the stress paths followed to reach to the current yield surface.

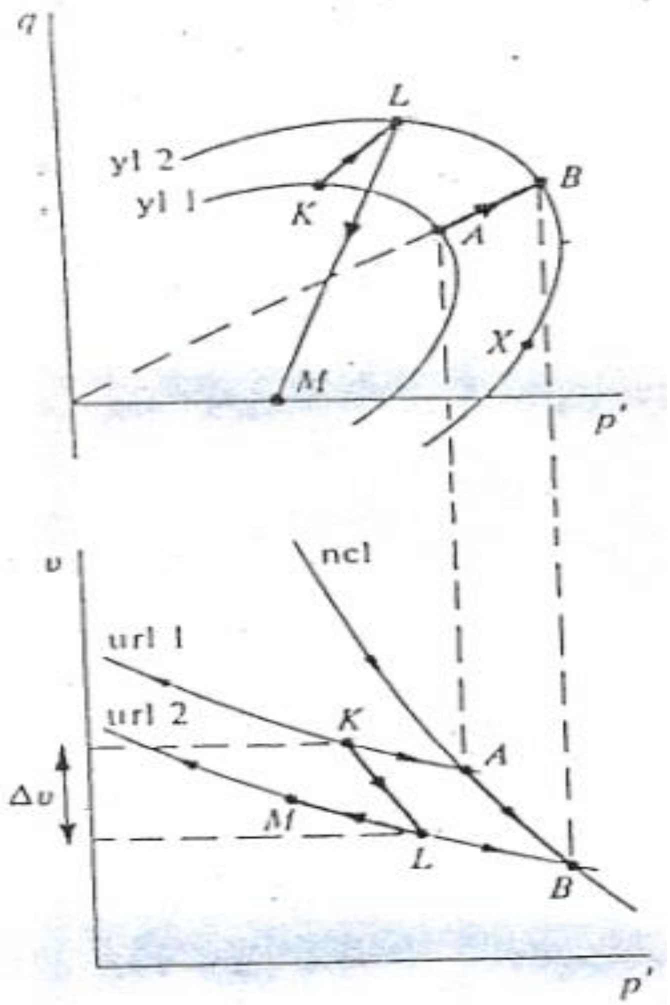
"Its Past Remains Hidden"

Elastic-Plastic Model

As shown , data from Winnipeg clay confirms these discussions.



Elastic-Plastic Model

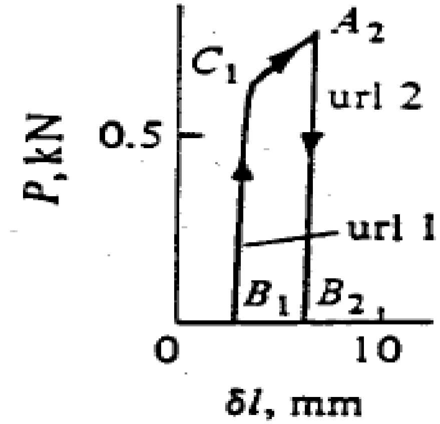


The total volume change that occurs during changes from point K to L is given by :

$$\Delta v = \Delta v^{e} + \Delta v^{p}$$

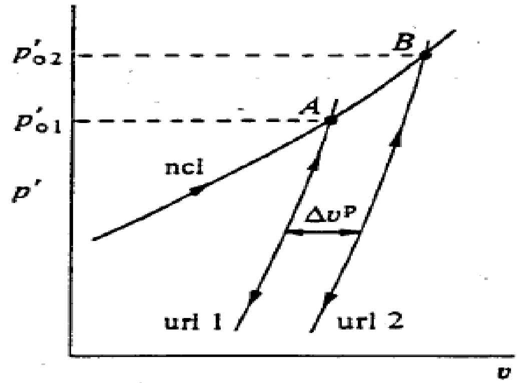
Elastic-Plastic Model

a) For the copper



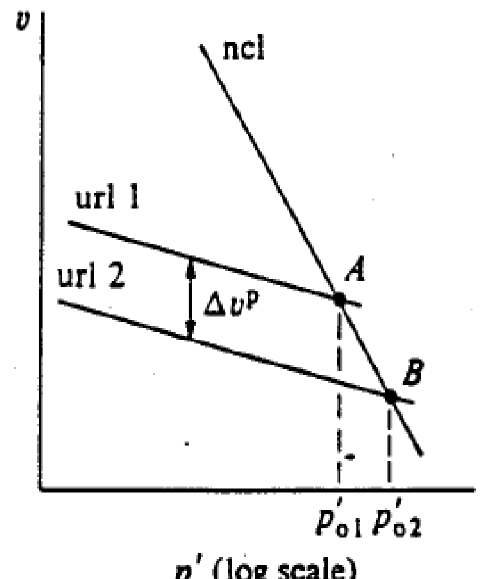
(a)

b) For soils



(b)

c) For soils (log scale)



(c)

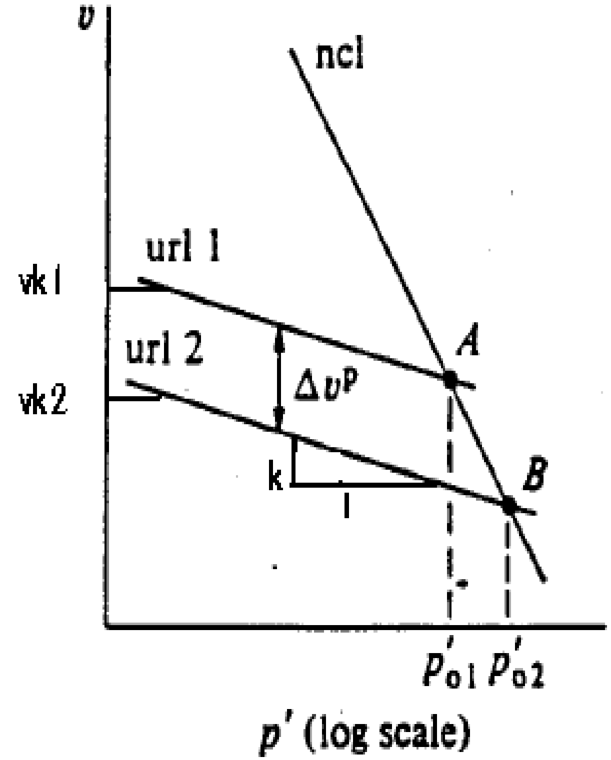
Elastic-Plastic Model

Each unloading curve correspond to a new yield point and the diff. of unloading curve gives the plastic volume change that has occurred during yielding to this new yielding point.

For UR1 : $v = v_{\kappa 1} - \kappa \ln p'$

For UR2 : $v = v_{\kappa 2} - \kappa \ln p'$

$$\Delta v^p = \Delta v_\kappa = v_{\kappa 2} - v_{\kappa 1}$$



Elastic-Plastic Model

Or in terms of mean pressure of point A and B ; and the normal compression line :

$$\Delta v^{p} = -\lambda \ln\left(\frac{p'_{o2}}{p'_{o1}}\right) + \kappa \ln\left(\frac{p'_{o2}}{p'_{o1}}\right)$$

$$= -(\lambda - \kappa) \ln\left(\frac{p'_{o2}}{p'_{o1}}\right)$$

Elastic-Plastic Model

In the limit :

$$\delta v^p = -(\lambda - \kappa)\frac{\delta p'_o}{p'_o}$$

In terms of strain :

$$\delta \varepsilon^p_p = (\lambda - \kappa)\frac{\delta p'_o}{v p'_o}$$

The elastic part was found before :

$$\delta \varepsilon^e_p = k\frac{\delta p'}{v p'}$$

In general ($\lambda - k$) (Plastic def. coef.) is about four times as big as k (Elastic def. coef.).

Elastic-Plastic Model

In general :

$$\delta v = \delta v^e + \delta v^p$$

or

$$\delta v = -\kappa \frac{\delta p'}{p'} - (\lambda - \kappa) \frac{\delta p'_o}{p'_o}$$

$$\delta \varepsilon_p = \delta \varepsilon_p^e + \delta \varepsilon_p^p$$

or

$$\delta \varepsilon_p = \kappa \frac{\delta p'}{v p'} + (\lambda - \kappa) \frac{\delta p'_o}{v p'_o}$$

Elastic-Plastic Model

Notes :

-Elastic volume changes occur whenever p' changes.

-Plastic volume changes occur whenever size of yield locus changes , specified by p'_0

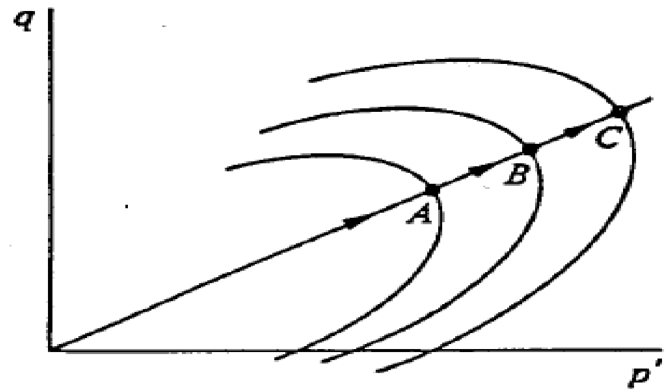
Elastic-Plastic Model

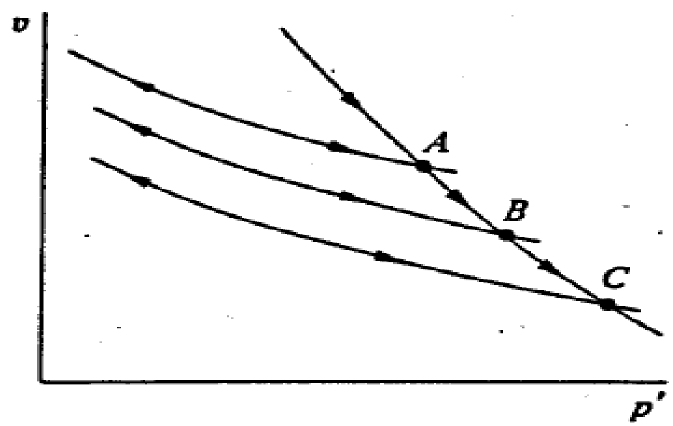
When the soil is being "normally compressed" then :

$$p' = p'_o$$

$$\delta v = -\lambda \frac{\delta p'}{p'}$$

$$\delta \varepsilon_p = \lambda \frac{\delta p'}{v p'}$$





Elastic-Plastic Model

Example :

PQ :

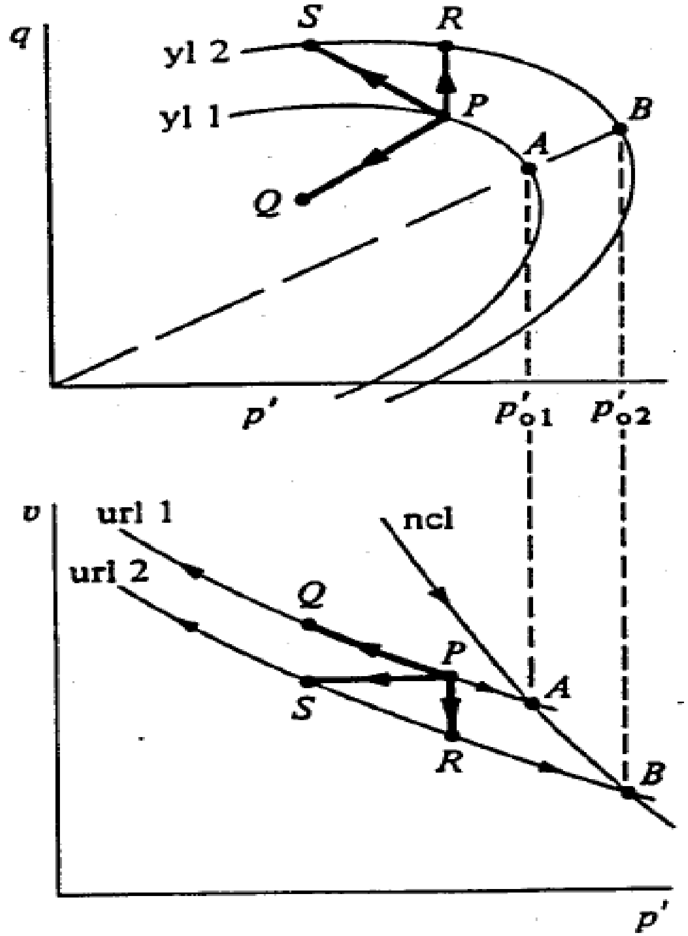
Interior of “Y1”

Elastic behavior :

$$\delta v^e = -\kappa \frac{\delta p'}{p'}$$

$$\delta \varepsilon_p^e = \kappa \frac{\delta p'}{v p'}$$

No change of p'_0



Elastic-Plastic Model

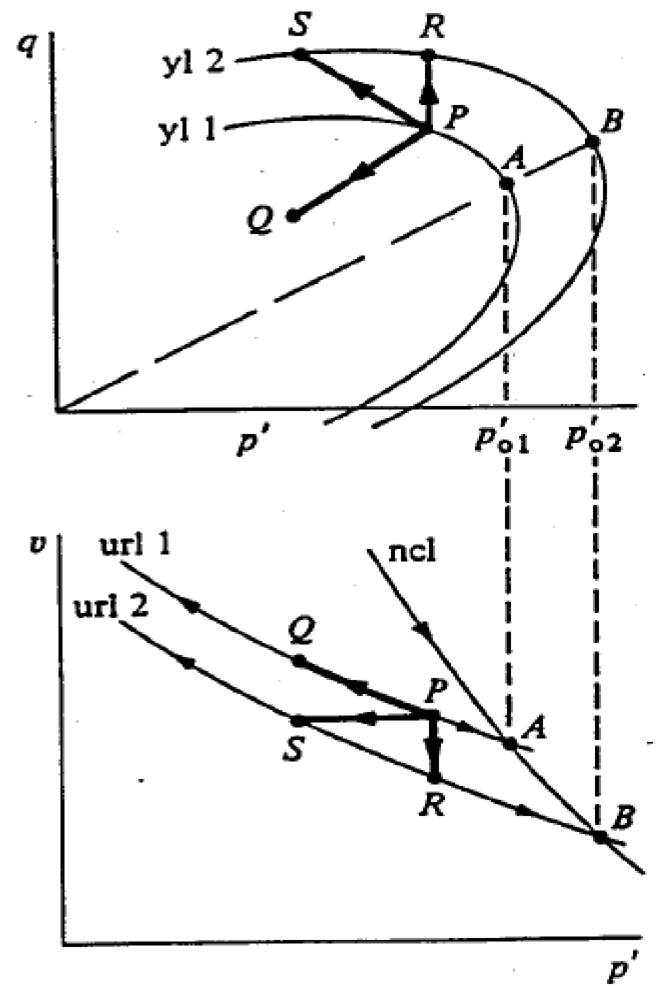
PR :

Vertical in q-p' space , "R" is on a new Yield locus "Y2" ➡ p'_{02}

➡ another reloading curve at point "B"

Since $\delta p' = 0$ ➡ $\delta \varepsilon_p^e = 0$, $\delta v^e = 0$

$$\delta \varepsilon_p^p = (\lambda - \kappa)\frac{\delta p'_0}{v p'_0}$$



Elastic-Plastic Model

PS :

No volume change : $\delta v = 0$

There is both elastic and plastic deformation

$$\delta \varepsilon_p = \delta \varepsilon_p^e + \delta \varepsilon_p^p = 0$$

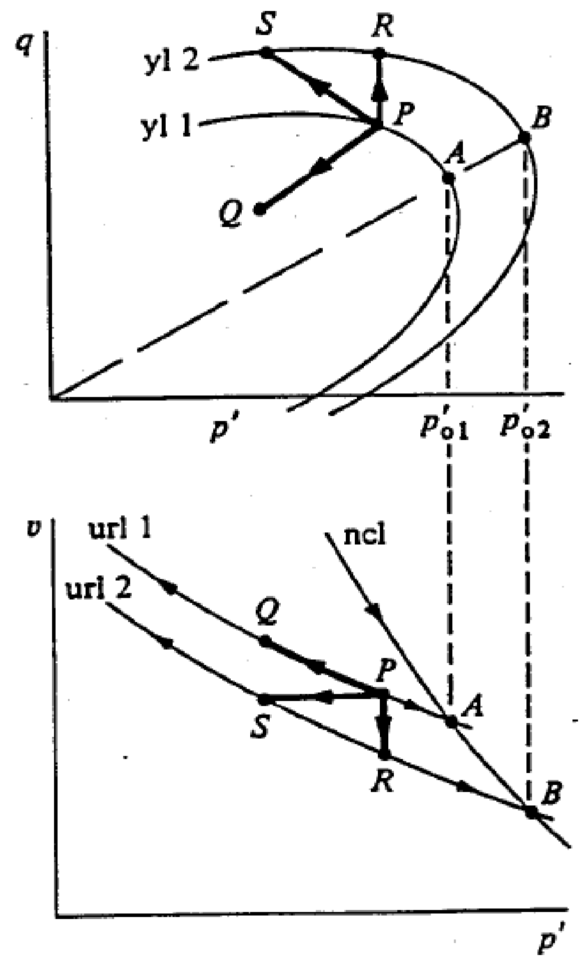
$$\delta \varepsilon_p^e = \kappa \frac{\delta p'}{v p'}$$

Since $\delta p' < 0$ ⇒ $\delta \varepsilon_p^e < 0$

$$\delta \varepsilon_p^p = (\lambda - \kappa) \frac{\delta p_o'}{v p_o'}$$

To have zero $\delta \varepsilon_p$ ⇒ $\delta \varepsilon_p^p > 0$

⇒ $\delta p_o' > 0$



Elastic-Plastic Model

Note 1 :

In undrained tests with no volume change , the individual elastic and plastic components may be different than zero.

Note 2 :

If the soil behaves elasticity (pure) then the condition of no volume change implies $\delta p' = 0$

Note 3 :

In general effective stress path in undrained tests does not have the same shape as the yield curve , since the yield curve has to change to produce plastic strain to balance the elastic volume change.

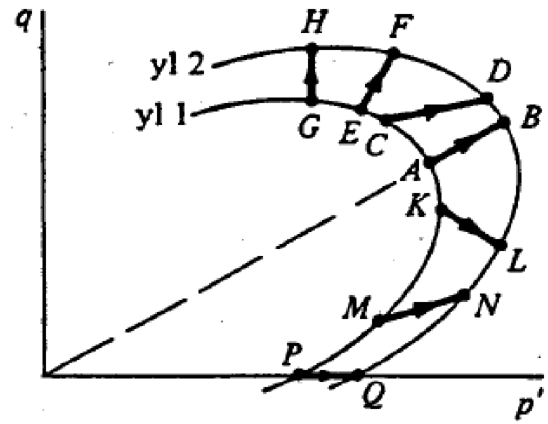
Elastic-Plastic Model

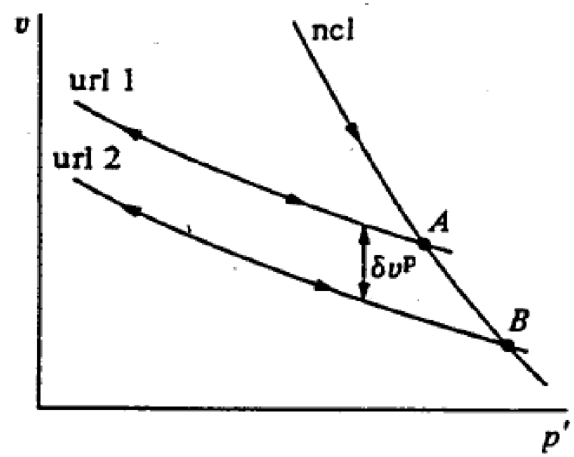
- Plastic Shear Strain :

Plastic volumetric change :

$$\delta v^p = -(\lambda - \kappa)\frac{\delta p'_o}{p'_o}$$

For all stress paths AB , CD , etc δv^p is the same since $\delta p'_0$ is the same and it only depends on the size of the yield locus.





Elastic-Plastic Model

Plastic shear strain ?

From the test described on the copper tube it was concluded that the directions of plastic strain increment vectors are not governed by the stress path but depend on the particular combination of stresses at the particular point at which the yield surface was reached .

Before going further in details , we discuss another example :

“Frictional Block”

Elastic-Plastic Model

Frictional Block :

A block sliding on arough surface :

Constant normal load “P” apply Qx

and increase till sliding occurs:

$$Q_x = \mu P$$

If both Qx and Qy is applied the

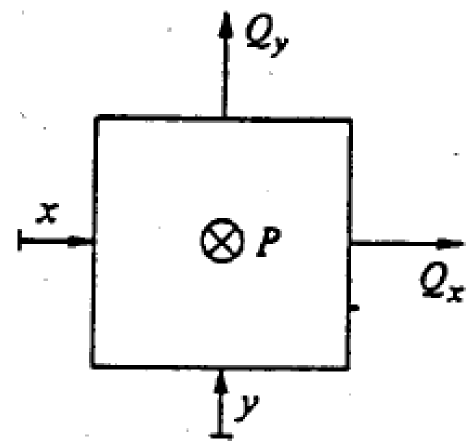
sliding occurs when the resultant

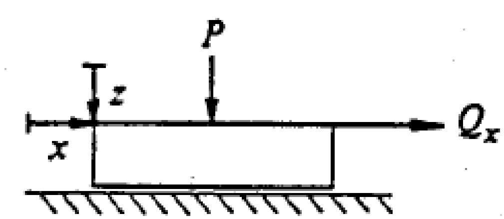
force = μp

$$\sqrt{Q_x^2 + Q_y^2} = \mu P$$

or

$$f = Q_x^2 + Q_y^2 - \mu^2 P^2 = 0$$





Elastic-Plastic Model

Last equation defines a sliding surface in P : Qx : Qy space.(a right circular cone on P axis)

In this equation :

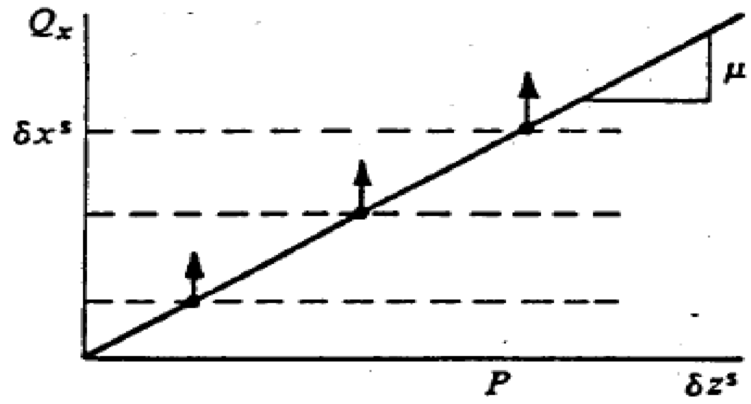
If $f<0$ → block remains still.

If $f=0$ → block slides.

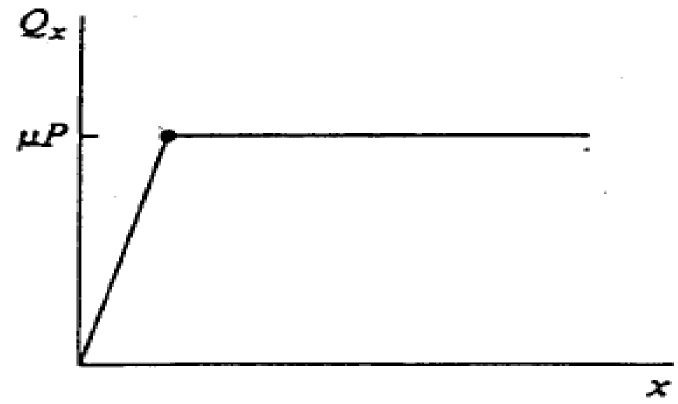
If $f>0$ → inadmissible.

Elastic-Plastic Model

On Qx-P plane :



Considering Qx and P only , there might be some elastic shear deformation of the block before sliding



Elastic-Plastic Model

-On P-Qx plane the sliding motion may be indicated by a vector of sliding component displacements :

δz^s , δx^s

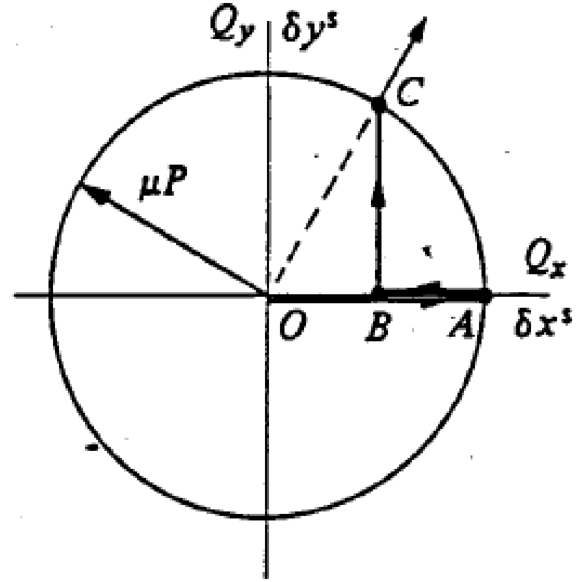
-Subscript "s" meaning sliding components , which means elastic components to the total displacement has been subtracted.(plastic remains)

-These vectors are all || to Qx axis since $\delta z^s = 0$

-If after sliding Qx is reduced to $\frac{1}{2}\mu p$ at constant P then Qy is applied , sliding will occur at $Q_y = \sqrt{3}\mu P/2$

Elastic-Plastic Model

Even though the sliding has occurred as a result of increasing Qy, the sliding occurs in the direction of resultant shear load so that the vector of sliding is always “**normal**” to the circular plane in Qx:Qy



Elastic-Plastic Model

So irrecoverable deformation occurs in both direction even though sliding was induced by increments in "y" direction.(depends on state of loading and on the path)

A second function "g" can introduced :

$$g = Q_x^2 + Q_y^2 - k^2 = 0$$

Elastic-Plastic Model

Right circular cylinder , the displacement vectors are vertical to this function , and :

$$\delta x^s = \chi \frac{\partial g}{\partial Q_x} = \chi 2 Q_x$$

$$\delta y^s = \chi \frac{\partial g}{\partial Q_y} = \chi 2 Q_y$$

$$\delta z^s = \chi \frac{\partial g}{\partial P} = 0$$

χ is a scalar multiplier

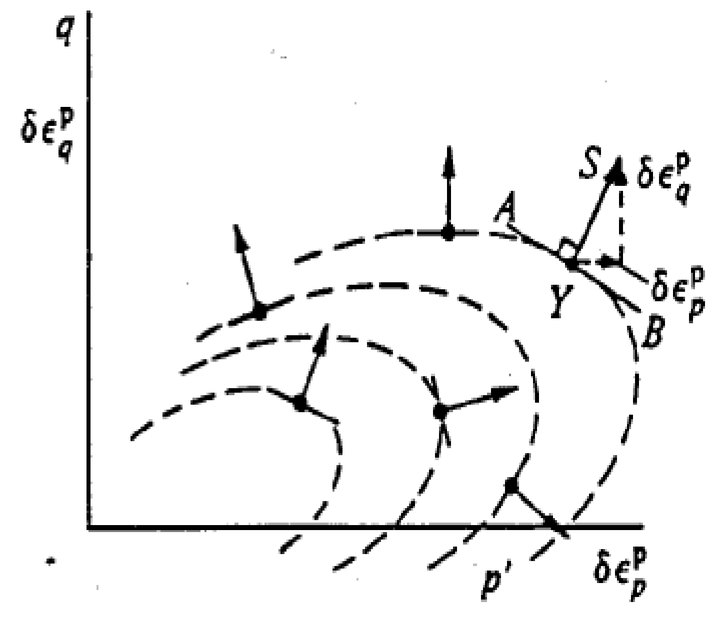
Elastic-Plastic Model

Plastic Potential :

Suppose yielding occurs at y , the yielding has two components of plastic deformation : Shear & Volumetric

A "plastic strain increment vector" is drawn at y. A normal to this vector at y is line "AB".

For each combination of stresses at yielding , a vector of plastic strains can be drawn with an orthogonal line to this vector.

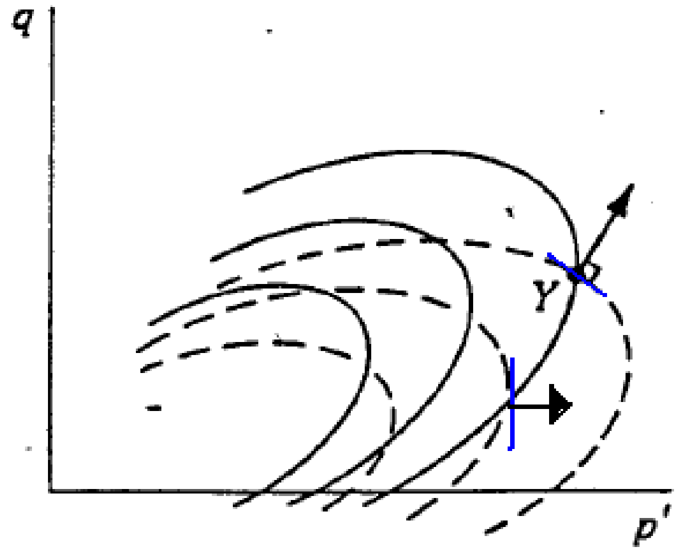


Elastic-Plastic Model

as enough data become available , these short normal lines from a family of curves to which plastic strain increment vectors are normal.

These curves are called : "Plastic Potential"

It is shown below :



Elastic-Plastic Model

Knowing the direction of plastic strain vectors , the relative magnitude of the two components of plastic strains are known.

knowing the volumetric part → known distortional or shear components

Elastic-Plastic Model

Normality Or Associated Flow :

If the plastic potential and the yield loci coincide , the two set of curves are identical as in the case of metal plasticity explained before.

But in the case of frictional block the two family of curves are not identical.

It is clearly an advantage if the shape of family of curves are the same since the number of functions needed for the plastic response is reduced by one.

When the yield surfaces and plastic potential surfaces are identical , the material is said to obey the postulate of "normality" , alternatively , the material is said to follow a law of "associated flow".

Elastic-Plastic Model

The yield data found by Graham et al (1983) are shown :

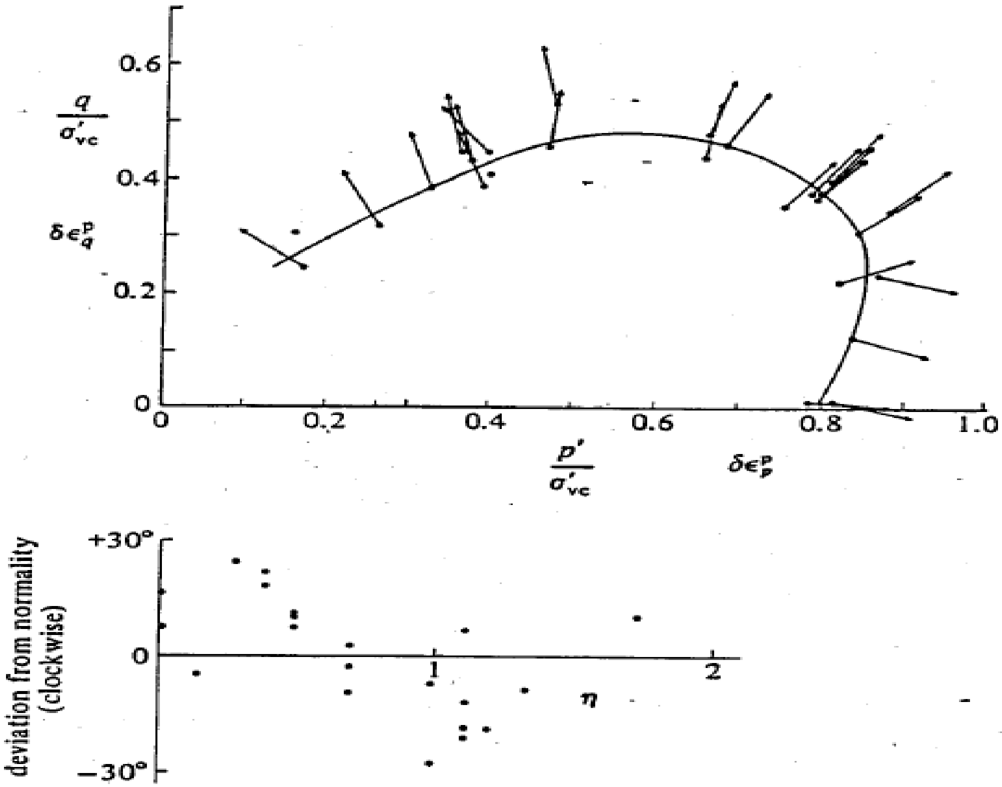
-plastic strain increments

are roughly vertical to the

yield locus. But actually it

varies 1 to about 20.

(roughly acceptable)

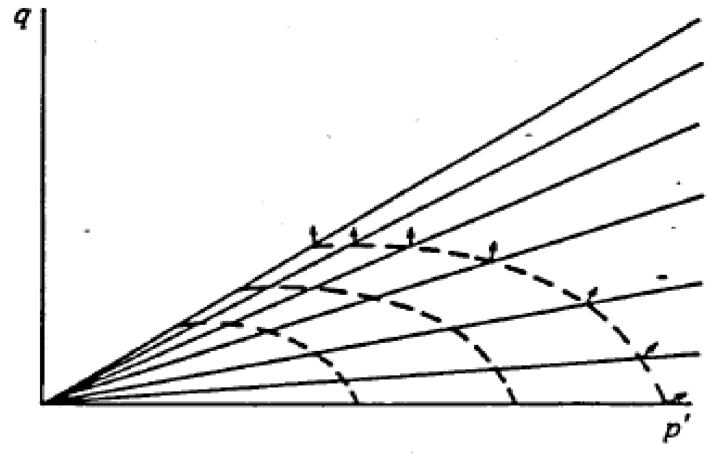


Elastic-Plastic Model

For sands , the proportion of normality is much less acceptable.

(Non associated flow)

Plastic potential lines and yield lines for sands (Ottawa sands) :



Elastic-Plastic Model

General Plastic Stress-Strain Relationship :

Yield locus : $f(p', q, p'_o) = 0$ (1)

p'_0 : indicates the size of any member of the family of the yield loci.

Plastic potential : $g(p', q, \zeta) = 0$ (2)

ξ : parameter controlingsize of plastic potential

Elastic-Plastic Model

The plastic strain increments vertical to "g" is given by :

$$\delta\varepsilon_p^p = \chi \frac{\partial g}{\partial p'} \qquad (3)$$

$$\delta\varepsilon_q^p = \chi \frac{\partial g}{\partial q} \qquad (4)$$

χ is a scalar multiplier

Suppose that the hardening , change in p'_0 (size) , is linked with both increments of volumetric and shear plastic strains :

$$\delta p'_o = \frac{\partial p'_o}{\partial \varepsilon_p^p}\delta\varepsilon_p^p + \frac{\partial p'_o}{\partial \varepsilon_q^p}\delta\varepsilon_q^p \qquad (5)$$

Elastic-Plastic Model

The differential form of the yield loci :

$$\frac{\partial f}{\partial p'}\delta p' + \frac{\partial f}{\partial q}\delta q + \frac{\partial f}{\partial p'_o}\delta p'_o = 0 \qquad (6)$$

Combining 3,4,5,6 : (omitting $\delta p'_0$)

$$\chi = -\left(\frac{\partial f}{\partial p'}\delta p' + \frac{\partial f}{\partial q}\delta q\right) \Big/ \frac{\partial f}{\partial p'_o}\left(\frac{\partial p'_o}{\partial \varepsilon^p_p}\frac{\partial g}{\partial p'} + \frac{\partial p'_o}{\partial \varepsilon^p_q}\frac{\partial g}{\partial q}\right)$$

Elastic-Plastic Model

Substituting X into 3 & 4 :

$$\begin{bmatrix} \delta\varepsilon_p^p \\ \delta\varepsilon_q^p \end{bmatrix} = \frac{-1}{\left[\dfrac{\partial f}{\partial p'_o}\left[\dfrac{\partial p'_o}{\partial \varepsilon_p^p}\dfrac{\partial g}{\partial p'} + \dfrac{\partial p'_o}{\partial \varepsilon_q^p}\dfrac{\partial g}{\partial q}\right]\right]} \begin{bmatrix} \dfrac{\partial f}{\partial p'}\dfrac{\partial g}{\partial p'} & \dfrac{\partial f}{\partial q}\dfrac{\partial g}{\partial p'} \\ \dfrac{\partial f}{\partial p'}\dfrac{\partial g}{\partial q} & \dfrac{\partial f}{\partial q}\dfrac{\partial g}{\partial q} \end{bmatrix} \begin{bmatrix} \delta p' \\ \delta q \end{bmatrix}$$

This is the final form , if : f=g ➤ associated flow

➤ Symmetric matrix

Elastic-Plastic Model

The elasto-plastic response needs :

-Elastic response (linear , nonlinear , isotropic , anisotropic)

-Yield surface (within which the response is elastic)

-Mode of plastic deformation (plastic potential)

-Hardening rule (changing size of the yield locus)

-Failure condition (Limiting surface , beyond which stress state can not ever pass)

Elastic-Plastic Model

H.W. :

#4.2 #4.5 #4.6 #4.7

Chapter # 5

Cam Clay

Cam Clay

- A particular model of soil behavior is presented to describe the yield locus and hardening rule

- It is a "volumetric" hardening model.

- Modified Cam Clay originally presented by Roscoe and Burland (1968).

- P' : q relevant to triaxial tests.

Cam Clay

- Elastic Properties :

  - Volumetric Strain

$$\delta\varepsilon_p^e = \kappa \frac{\delta p'}{v p'}$$

  - Distortional Strain

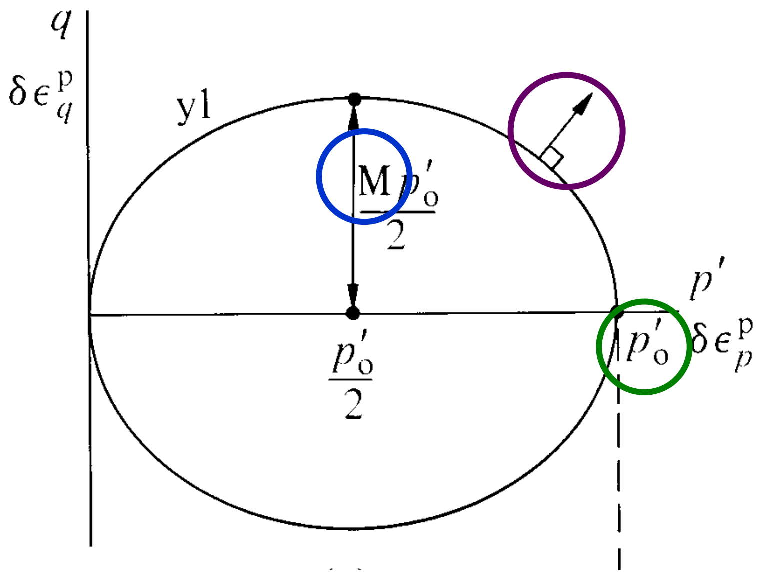
$$\delta\varepsilon_q^e = \frac{\delta q}{3G'}$$

Cam Clay

- Yield Locus :
- A simple shape for yield locus: ellipse
- shape M
- size p'_o
- plastic volumetric strains linked with change in p'_o

$$\frac{p'}{p'_o} = \frac{M^2}{M^2 + \eta^2} \qquad \eta = q/p'$$

$$\sigma'_1, \sigma'_2, \sigma'_3$$



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Equation (1) may be rewritten as :

Yield :

$$f = q^2 - M^2[p'(p'_o - p')] = 0 \qquad (3)$$

Having Associated Flow Rule , g≡f :

Plastic potential :

$$g = f = q^2 - M^2[p'(p'_o - p')] = 0 \qquad (4)$$

$$\frac{\delta\varepsilon_p^p}{\delta\varepsilon_q^p} = \frac{\partial g/\partial p'}{\partial g/\partial q} = \frac{M^2(2p' - p'_o)}{2q} = \frac{M^2 - \eta^2}{2\eta} \qquad (5)$$

Cam Clay

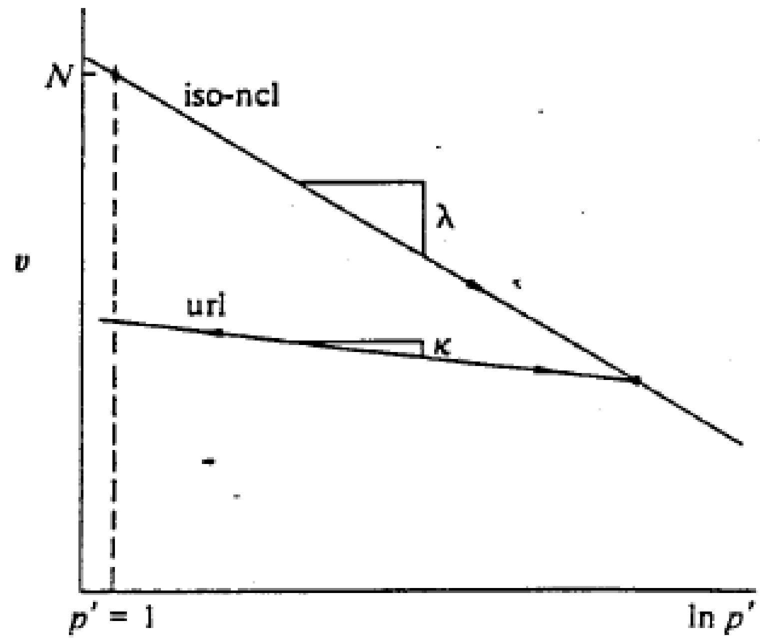
Hardening :

$$v = N - \lambda \ln p'_o \qquad (6)$$

$$\delta\varepsilon_p^p = [(\lambda - \kappa)/v]\,\frac{\delta p'_o}{p'_o} \qquad (7)$$

$$\frac{\partial p'_o}{\partial \varepsilon_p^p} = \frac{v p'_o}{\lambda - \kappa}$$

$$\frac{\partial p'_o}{\partial \varepsilon_q^p} = 0$$



(8)

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Now the model's description is complete and the final matrix from the plastic deformation as obtained previously can be formed :

$$\begin{bmatrix} \delta\varepsilon_p^p \\ \delta\varepsilon_q^p \end{bmatrix} = \frac{(\lambda-\kappa)}{vp'(M^2+\eta^2)} \begin{bmatrix} (M^2-\eta^2) & 2\eta \\ 2\eta & 4\eta^2/(M^2-\eta^2) \end{bmatrix} \begin{bmatrix} \delta p' \\ \delta q \end{bmatrix} \qquad (10)$$

And the elastic part :

$$\begin{bmatrix} \delta\varepsilon_p^e \\ \delta\varepsilon_q^e \end{bmatrix} = \begin{bmatrix} \kappa/vp' & 0 \\ 0 & 1/3G' \end{bmatrix} \begin{bmatrix} \delta p' \\ \delta q \end{bmatrix} \qquad (9)$$

Cam Clay

NOTE :The determinant of Eq.10 is zero , indicating dependence of $\delta\varepsilon_p^p$, $\delta\varepsilon_q^p$

this is clear since their ratio was shown to depend on the stress ratio (Eq.5) and not on the stress "increments".

NOTE :The two sets of Eq.10 , 9 are quite general and valid for all stress paths in p'-q space.

Cam Clay

Application :

$$stress - state : \begin{cases} A(p' : q) \\ M \end{cases} \rightarrow \quad p'_{0A} \quad using \;\; Eq(1)$$

$$stress \;\; increment : \;\; (\delta p' , \delta q)$$

$$\rightarrow \;\; B(p' + \delta p' \;,\;\; q + \delta q) \;\; \rightarrow \quad p'_{0B} \quad using \, Eq.(1)$$

Compare p'_{0A} , p'_{0B} to decide whether elastic or plastic deformation occurs ; then using appropriate equations to determine their magnitudes.

Cam Clay

Proof of Eq.(5) :

$$\frac{\delta\varepsilon_p^p}{\delta\varepsilon_q^p} = \frac{M^2(2p' - p'_0)}{2q}$$

$$from \quad Eq.1 \quad \rightarrow \quad p'_0 = \frac{p'(M^2 + \eta^2)}{M^2}$$

$Replace \ p'_0$:

$$\Rightarrow \frac{\delta\varepsilon_p^p}{\delta\varepsilon_q^p} = \frac{M^2\left(2p' - \frac{p'(M^2+\eta^2)}{M^2}\right)}{2q}$$

$$= \frac{2p'M^2 - p'M^2 - p'\eta^2}{2q}$$

$$= \frac{p'(M^2 - \eta^2)}{2\eta p'} = \frac{M^2 - \eta^2}{2q}$$

Cam Clay

Proof of Equation 5.4b of the book :

$$df = \frac{\partial f}{\partial q}\delta q + \frac{\partial f}{\partial p'}\delta p' + \frac{\partial f}{\partial p'_0}\delta p'_0 = 0$$

$$= 2q\delta q - M^2(p'_0 - 2p')\delta p' - M^2 p'\delta p'_0 = 0$$

From Eq. of Ellipse :

$$p'_0 = \frac{p'(M^2 + \eta^2)}{M^2} \quad , \quad M^2 = \frac{p'(M^2 + \eta^2)}{p'_0}$$

Replace for p'_0 , M^2 :

$$df = 2\eta p'\delta q - p'(\eta^2 - M^2)\delta p' - \frac{p'^2(M^2 + \eta^2)}{p'_0}\delta p'_0 = 0$$

$$\left(\frac{2\eta}{M^2 + \eta^2}\right)\frac{\delta q}{p'} + \left(\frac{M^2 - \eta^2}{M^2 + \eta^2}\right)\frac{\delta p'}{p'} - \frac{\delta p'_0}{p'_0} = 0$$

Cam Clay

- Prediction :

a) Conventional CD-Test

$$q = \sigma_1' - \sigma_3'$$

$$p' = \frac{1}{3}(\sigma_1' + \sigma_2' + \sigma_3') = \frac{1}{3}(\sigma_1' + 2\sigma_3')$$

U=0

since $\delta\sigma_3 = \delta\sigma_3' = 0$ ⇒ $\delta q = 3\delta p'$

Cam Clay

Increment of loading BC

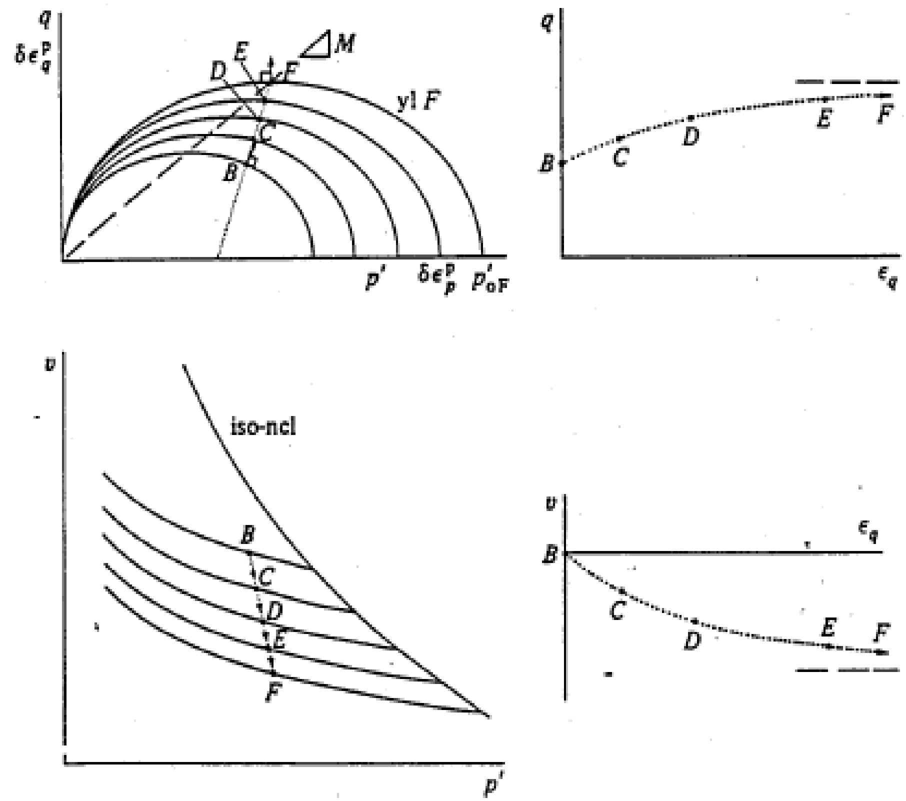
volume change :

$$\delta v_{BC} = \delta v^e_{BC} + \delta v^p_{BC}$$

$$\delta \varepsilon^p_{pBC} = \frac{-\delta v^p_{BC}}{v_B}$$

Knowing λ & k so :

$\delta \varepsilon^p_p$ an be calculated.



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-The ratio $\frac{\delta \varepsilon_p^p}{\delta \varepsilon_q^p}$ can be found from the normal direction to the yield locus → $\delta \varepsilon_q^p$

-further increments of load may be applied till point "F" where the ratio of "plastic vol." & "shear def." approaches infinity.

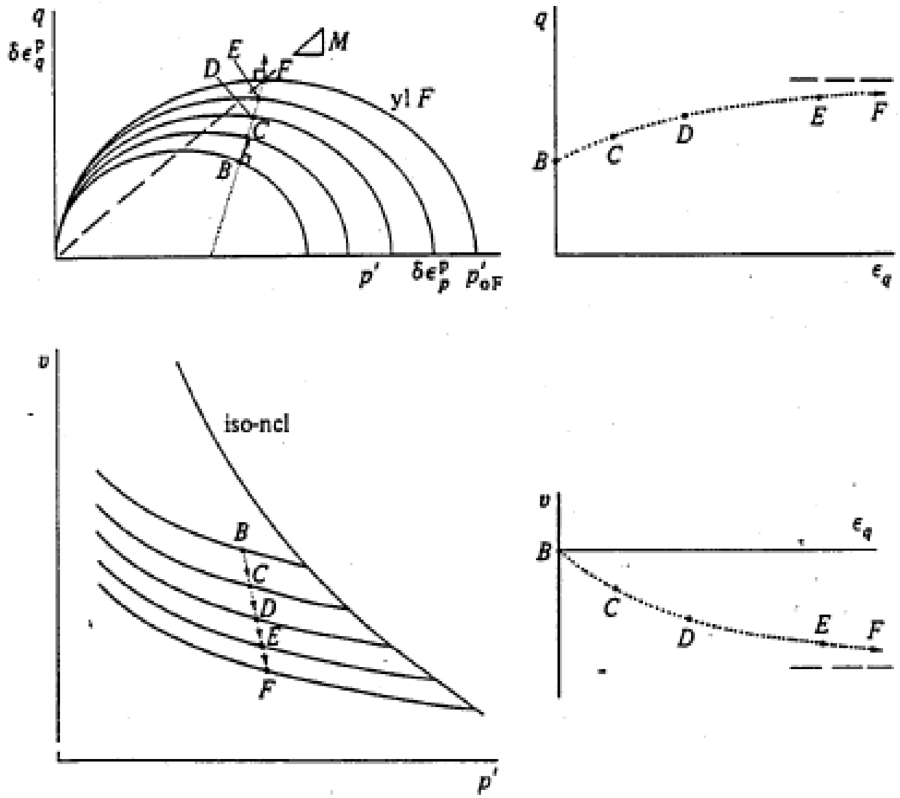
At this point :

$$\frac{q}{p'} = \eta = M, \quad p' = \frac{p'_{oF}}{2} \;\Rightarrow\; \frac{\delta \varepsilon_p^p}{\delta p'} = 0, \quad \frac{\delta \varepsilon_q^p}{\delta q} = \infty$$

Since $\delta \varepsilon_p^p = 0$ → "No further change in the size of yield locus"

Cam Clay

"plastic shear strain continues at constant effective stress at point F "



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These behaviors may be followed using the equation derived previously :

$$\frac{\delta\varepsilon_p^p}{\delta\varepsilon_q^p} = \frac{M^2 - \eta^2}{2\eta}$$

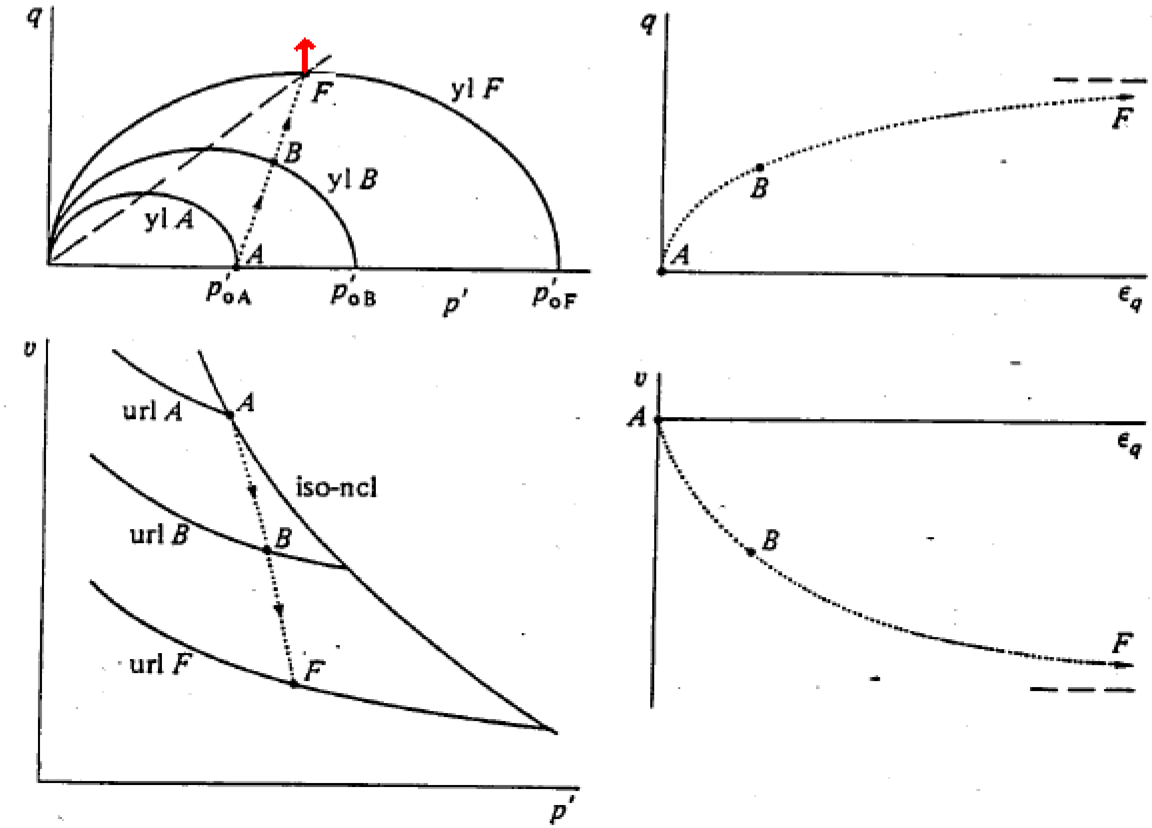
$$\therefore \quad as \qquad \eta \to M \quad \Rightarrow \quad \frac{\delta\varepsilon_p^p}{\delta\varepsilon_q^p} \to 0$$

Now considering the route by which point B was reached :

a) The soil has been normally consolidated to point A in the figure shown. In this case plastic deformations occur as soon as the drained compression begins , since the yield locus has to expand

Cam Clay

- Normally consolidated clay :



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b) if the soil has been compressed to B (isotropically) and unloaded to A ; such that it is lightly over consolidated :

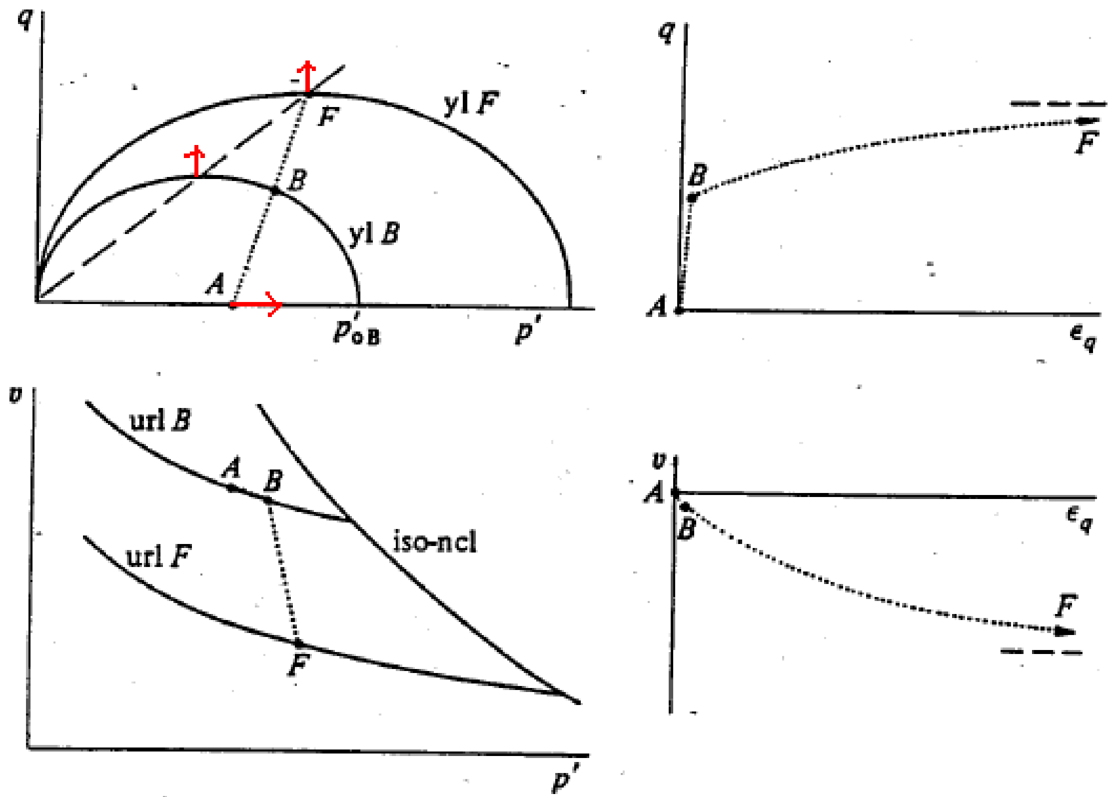
-in this case , the current yield locus passes through poit B , and drained loading till intersection of the yield locus is "purely elastic".

-at point B , there is a sharp drop in stiffness , where plastic deformations start.

-same behavior in volume change

Cam Clay

- Lightly over consolidated clay



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c) in the case "Heavily overconsolidated" ; if the soil is compressed to "K" and then unloaded to "P" ; we have :

-response is elastic till point Q on the yield locus "Yl Q".

-since Q is on the left of the top of yield locus :

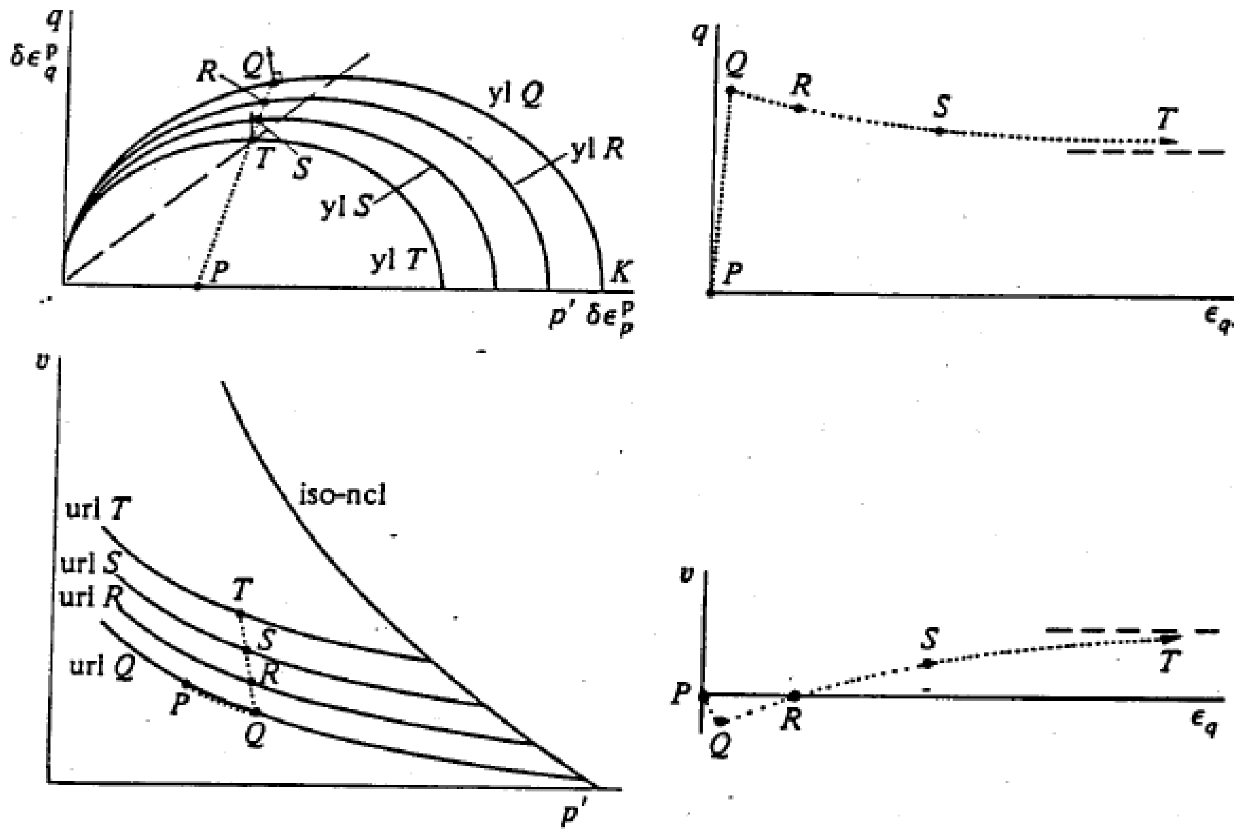
$$\Rightarrow \quad \delta\varepsilon_p^p < 0$$

From $\delta\varepsilon_p^p = \frac{(\lambda - k)}{v} \frac{\delta p_0'}{p_0'} \quad \Rightarrow \quad \delta p_0' < 0$

"this means contraction of the yield locus"

Cam Clay

- Heavily overconsolidated



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Note :

The ESP retreats back as shown in last figure toward point P (R , S , …) till the direction of plastic strain vector becomes vertical : $\eta = M$

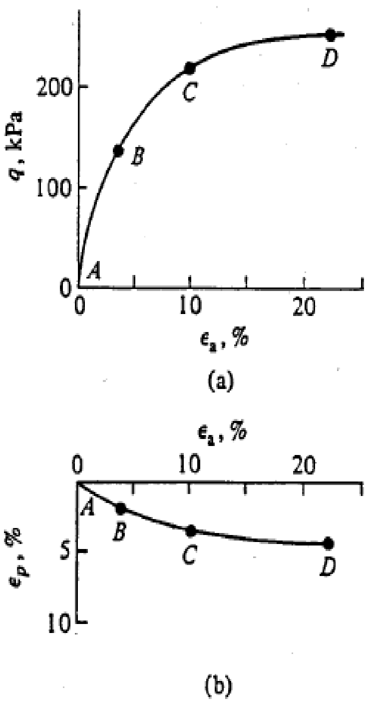
The plastic shear strain continues without change in the yield locus.

in this case ; plastic softening occurs instead of "work-hardening"

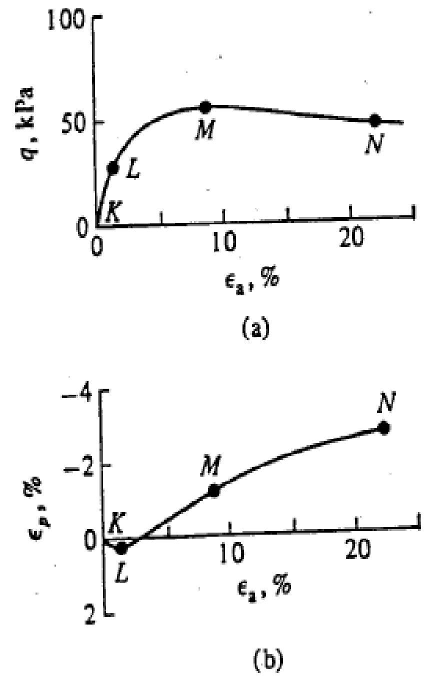
Cam Clay

- Experimental Results :

Conventional drained triaxial compression test on normally compressed Weald clay [$\sigma'_r = 207$ kPa (30 lbf/in^2)]: (a) deviator stress q and axial strain ε_a; (b) volumetric strain ε_p and axial strain ε_a (after Bishop and Henkel, 1957).



Conventional drained triaxial compression test on heavily overconsolidated Weald clay [$\sigma'_r = 34$ kPa (5 lbf/in.^{2})]: (a) deviator stress q and axial strain ε_a; (b) volumetric strain ε_p and axial strain ε_a (after Bishop and Henkel, 1957).



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- Prediction :

b) Undrained Triaxial Behavior

$$\delta\varepsilon_p = \delta\varepsilon_p^p + \delta\varepsilon_p^e = 0$$

$$\Rightarrow \quad k\frac{\delta p'}{p'} + (\lambda - k)\frac{\delta p'_0}{p'_0} = 0 \qquad (1) \qquad \textit{Link between } \delta p' \;\&\; \delta p'_0$$

from eq.(1) and shape of yield locus given by :

$$\frac{\delta p'}{p'} + \frac{2\eta\delta\eta}{M^2 + \eta^2} - \frac{\delta p'_0}{p'_0} = 0 \qquad (2)$$

Cam Clay

Note :

yield Surface :

$$\frac{p'}{p'_0} = \frac{M^2}{M^2 + \eta^2}$$

$$\frac{-p'\delta p'_0 + p'_0 \delta p'}{p'^2_0} = \frac{-M^2 \ 2\eta d\eta}{(M^2 + \eta^2)^2} = -\frac{p'}{p'_0} \left(\frac{2\eta \ \delta\eta}{M^2 + \eta^2}\right)$$

$$\delta p' - p' \frac{\delta p'_0}{p'_0} = -p' 2\eta\delta\eta / (M^2 + \eta^2)$$

$$\frac{\delta p'}{p'} + \frac{2\eta\delta\eta}{M^2 + \eta^2} - \frac{\delta p'_0}{p'_0} = 0$$

Cam Clay

$$(1)\ \&\ (2):\qquad -\frac{\delta p'}{p'} = \frac{\lambda - k}{\lambda}\frac{2\eta}{M^2 + \eta^2}d\eta \qquad (3)$$

$$Integration:\qquad \int_{p_i'}^{p'} (\,)\delta p \quad , \quad \int_{\eta_i}^{\eta} d\eta$$

$$\frac{p_i'}{p'} = \left(\frac{M^2 + \eta^2}{M^2 + \eta_i^2}\right)^{\Lambda} \qquad (4)$$

$$where \quad \Lambda = \frac{\lambda - k}{\lambda}$$

$$p_i', \eta_i \quad : \quad Initial\ effective\ stress - state$$

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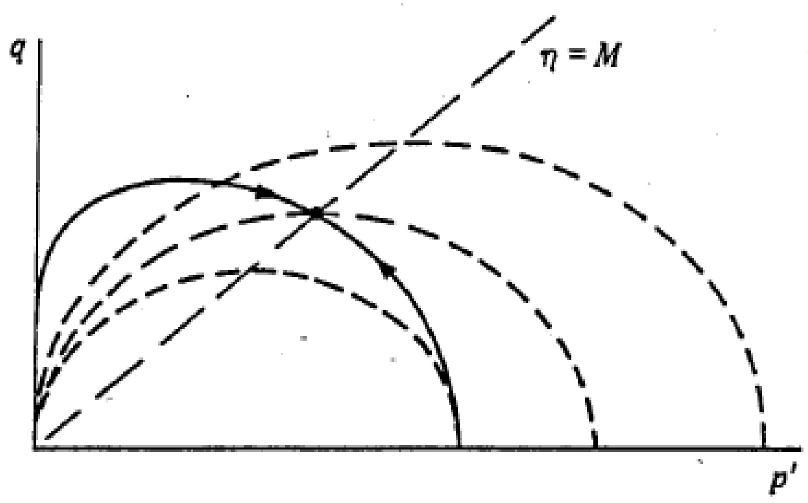
- Equation (4) defines the shape of the undrained effective stress path. (provided yielding is occurring)



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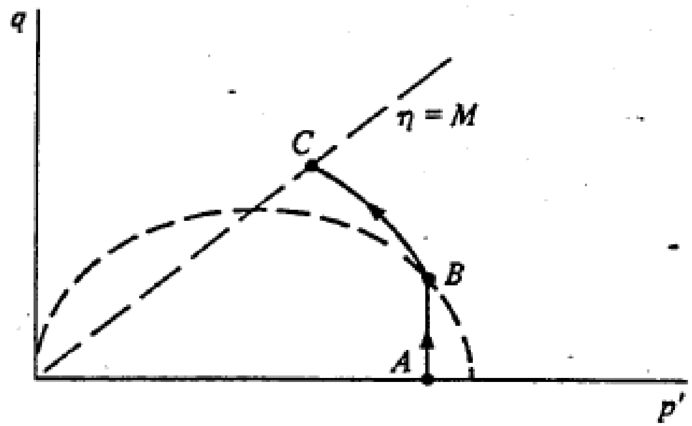
- If the ESP is within the current yield surface :

→ Elastic behavior

→ $\delta p' = 0 \quad (for\ \delta\varepsilon_p = 0)$



$AB : Elastic \quad ; \quad BC : Elastic + plastic$

Cam Clay

$from\ eq.(1)\ sign\ of\ \delta p'\ ,\delta p'_0\ should\ be\ opposite:$

$$\therefore\ if\quad \eta < M\ ,\ \delta\varepsilon_p^p > 0\quad \rightarrow\quad \delta p'_0 > 0$$

$$then\quad \delta p' < 0$$

$$\therefore\ if\quad \eta > M\ ,\ \delta\varepsilon_p^p < 0\quad \rightarrow\quad \delta p'_0 < 0$$

$$then\quad \delta p' > 0$$

Cam Clay

- Note :

ESP in the compression triaxial requires NO knowledge of TSP.

The same ESP is followed for any triaxial compression total stress path.

The only difference is different PWP that develops.

$$\delta u = \delta p - \delta p'$$

$$\delta u = \delta p + a\delta q \quad , \quad a = \frac{-\delta p'}{\delta q} \qquad (1)$$

$$\therefore \; a = slope\ of\ \ ESP$$

Cam Clay

$Triaxial\ \ Test\ :\qquad TSP\ :\ \ \delta q = 3\delta p$

If Elastic behavior $\qquad \delta p' = 0\ \ ,\qquad \rightarrow a = 0$

$$\delta u = \delta p = \frac{\delta q}{3}$$

If plastic behavior

$$\delta q = \eta \delta p' + p' \delta \eta \qquad (2)$$

$$previously\ :\ -\frac{\delta p'}{p'} = \frac{\lambda - k}{\lambda} \frac{2\eta}{M^2 + \eta^2} \delta \eta \qquad (3)$$

Cam Clay

$$(1),(2),(3) \quad \rightarrow \quad \boxed{a = \frac{2(\lambda - k)\eta}{\lambda(M^2 + \eta^2) - 2(\lambda - k)\eta^2}}$$

proof :

$$-\frac{\delta p'}{p'} = \frac{\lambda - k}{\lambda} \frac{2\eta}{M^2 + \eta^2} \delta\eta$$

$$-\delta p' = \frac{\lambda - k}{\lambda} \frac{2\eta}{M^2 + \eta^2} p' \delta\eta$$

Cam Clay

$$\delta q = \eta \delta p' + p' \delta \eta = \eta \delta p' + \frac{-\delta p'}{\frac{\lambda - k}{\lambda} \frac{2\eta}{M^2 + \eta^2}}$$

$$\delta q = \left[\eta - \frac{\lambda (M^2 + \eta^2)}{2\eta(\lambda - k)} \right] \delta p'$$

$$a = -\frac{\delta p'}{\delta q} = -1 \Big/ \left[\frac{2\eta^2(\lambda - k) - \lambda(M^2 + \eta^2)}{2\eta(\lambda - k)} \right]$$

$$a = \frac{2\eta(\lambda - k)}{\lambda(M^2 + \eta^2) - 2\eta^2(\lambda - k)}$$

Cam Clay

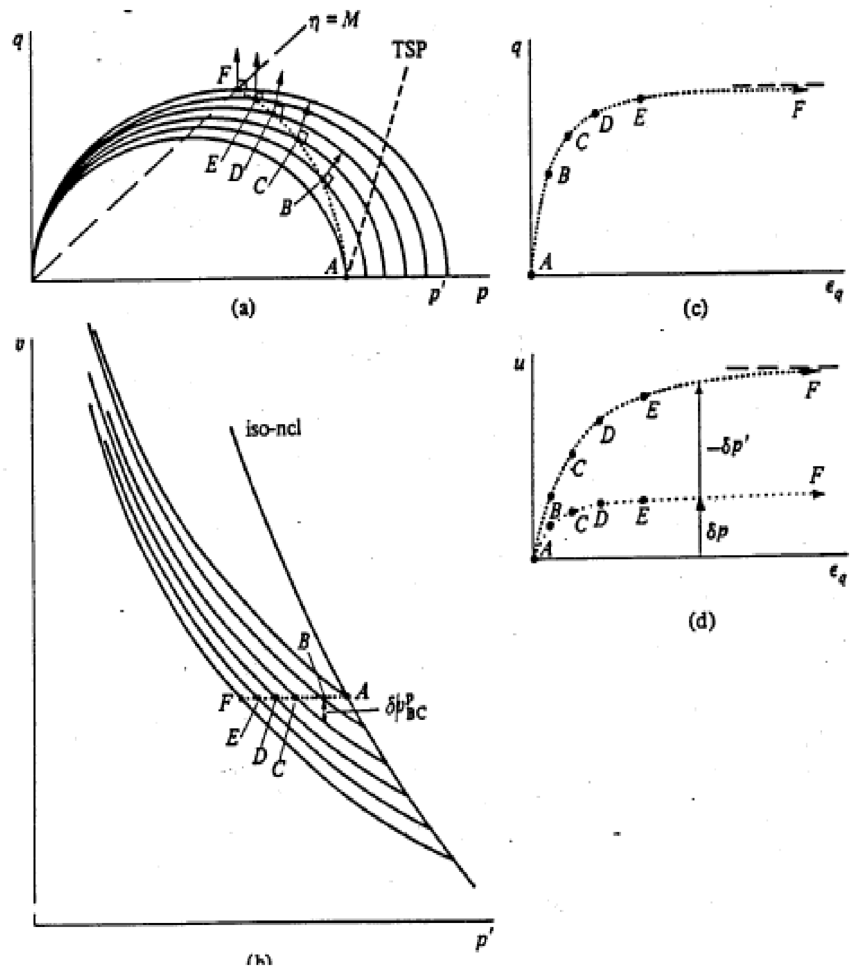
$since\ \ \delta v = 0 \quad \rightarrow \quad points\ \ B, C, \ldots, F\ \ in\ \ Fig.b$

$\rightarrow \quad points\ \ B, C, \ldots, F\ \ in\ \ Fig.a$

$\rightarrow \quad ESP$

$Fig.c \quad \rightarrow \quad \delta u$

δu is divided to two parts showing contribution of total stress δp , and from volume change $\delta p'$.



Cam Clay

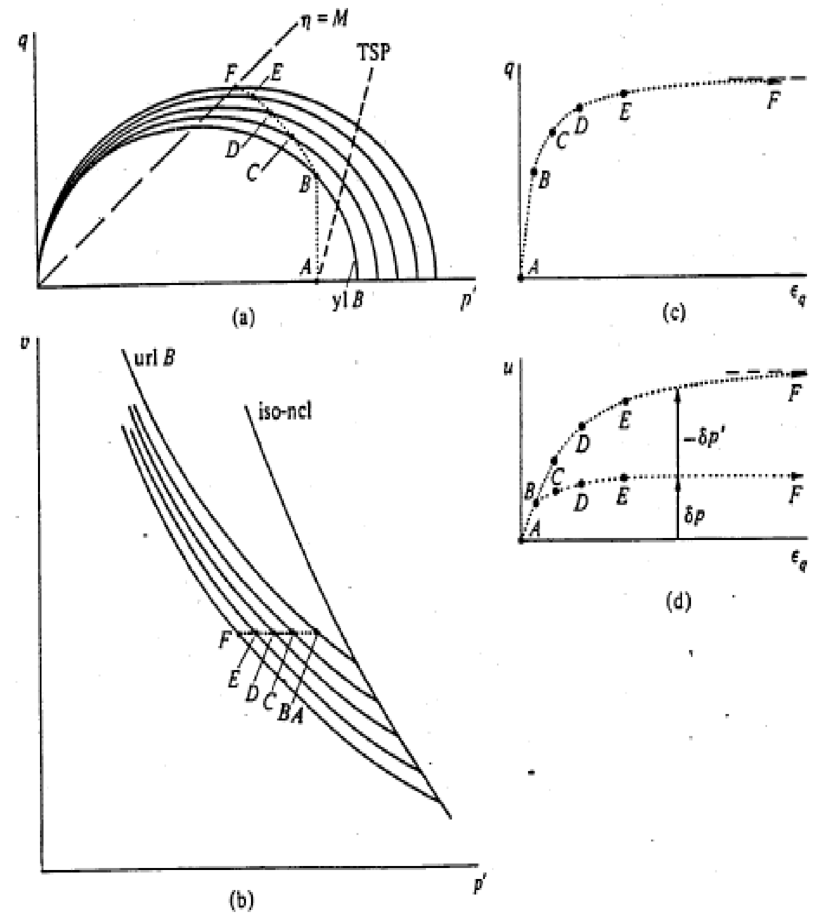
Lightly overconsolidated :

Elastic plane AB

Fig b : there is no change in position till plastic vol change occur.

“AB” associated only with Elastic shear Strain :

$$\delta p' = 0 \quad , \quad \delta u = \delta p$$



Cam Clay

- Heavily overconsolidated :

$$Elastic\ :\quad PQ\ ,\quad \delta p' = 0$$

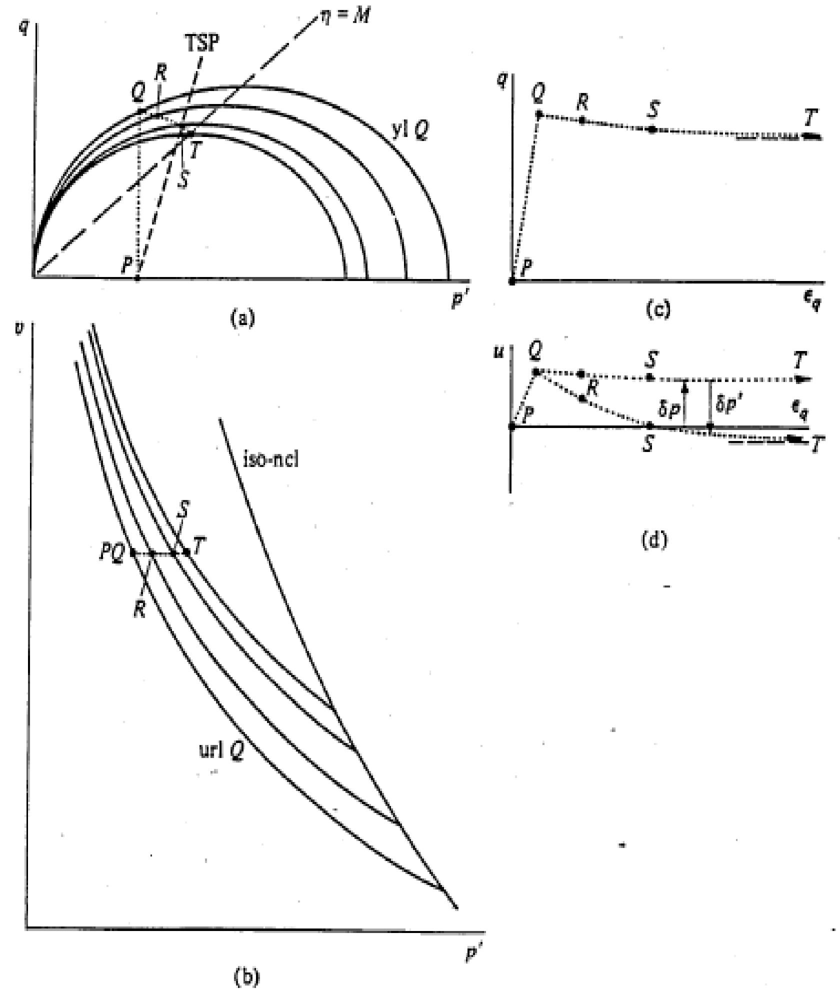
$$\eta > M \quad \rightarrow \quad shrinking\ \ of\ \ yield\ \ locus$$

$$\rightarrow \quad \delta p' > 0$$

Max PWP at yield , will be

reduced substentially.

$$a < 0 \quad \rightarrow \quad \delta u < 0$$



Cam Clay

- These figures confirms predicted behavior , (Tests on real soils) :

Fig. 5.18 Conventional undrained triaxial compression test on normally compressed Weald clay [$\sigma'_c = 207$ kPa (30 lbf/in.^{2})]: (a) deviator stress q and triaxial shear strain ϵ_q; (b) pore pressure change Δu and triaxial shear strain ϵ_q (after Bishop and Henkel, 1957).

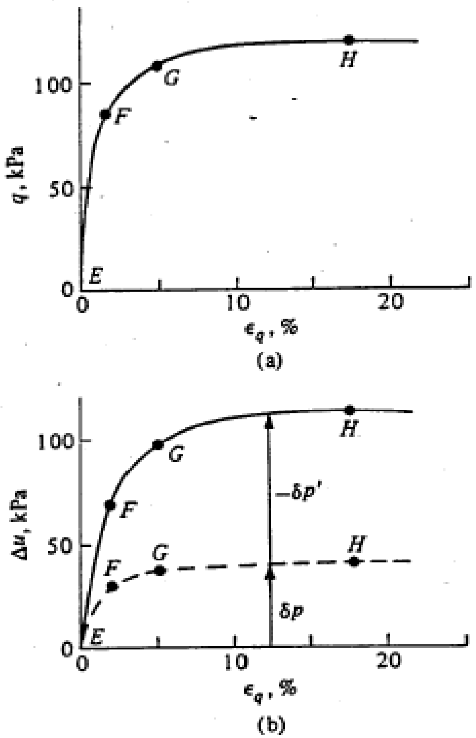
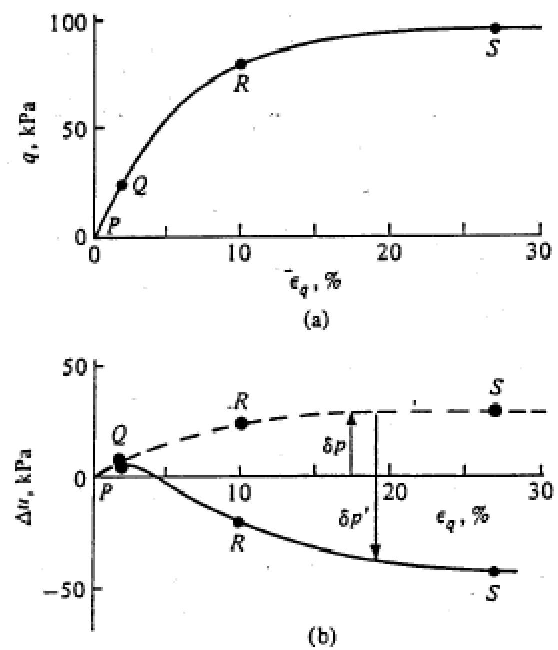


Fig. 5.19 Conventional undrained triaxial compression test on heavily overconsolidated Weald clay [$\sigma'_c = 34$ kPa (5 lbf/in.^{2})]: (a) deviator stress q and triaxial shear strain ϵ_q; (b) pore pressure change Δu and triaxial shear strain ϵ_q (after Bishop and Henkel, 1957).



Cam Clay

- Conclusion :

  The simple model is capable of predicting any stress path behavior.

  Parameters :

  $k \quad G'$

  $M \quad \lambda \quad N$

Cam Clay

- HWs :

#5.1 , #5.4

Chapter #6

Critical State

Critical state

- Critical state is defined as the condition of perfect plasticity :

$$\frac{\partial p'}{\partial \varepsilon_q} = \frac{\partial q}{\partial \varepsilon_q} = \frac{\partial v}{\partial \varepsilon_q} = 0$$

- The corresponding effective stress ratio :

$$\eta_{cs} = \frac{q_{cs}}{p'_{cs}} = M$$

Critical state

As seen before , the ultimate point of effective stress path for both drained and undrained condition , is $\eta = M$ on top of the yield loci the critical state line may be reached from both sides , ie

$$\eta < M \;,\; \eta > M \quad \rightarrow \quad \eta_{cs} = M$$

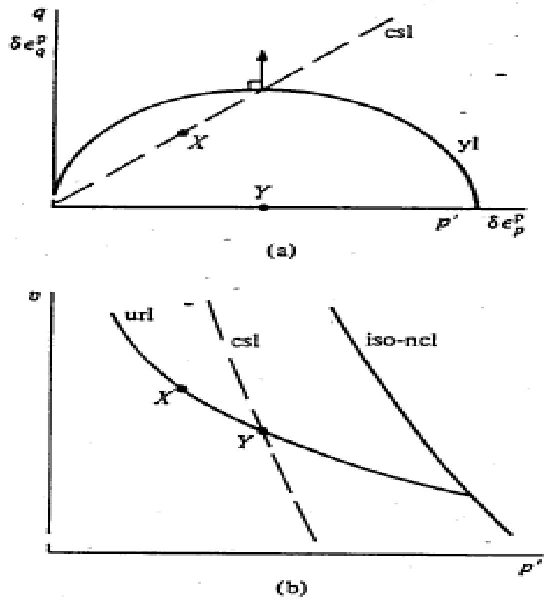
Critical state

- Note :

Critical state is reached on $\eta = M$, provided plastic deformations are occurring.

$\eta = M$ may be reached elastically !

Fig. 6.2 Points X and Y inside current yield locus and not at critical state.



Critical state

- The cam-clay yield loci :

$$\frac{p'}{p'_0} = \frac{M^2}{M^2 + \eta^2}$$

Its size is controlled by p'_0 , shape (top of yield locus) by "M".

Top

$$p' = p'_{cs} = \frac{p'_0}{2}$$

$$\eta = M \text{ , or } \quad q_{cs} = M p'_{cs}$$

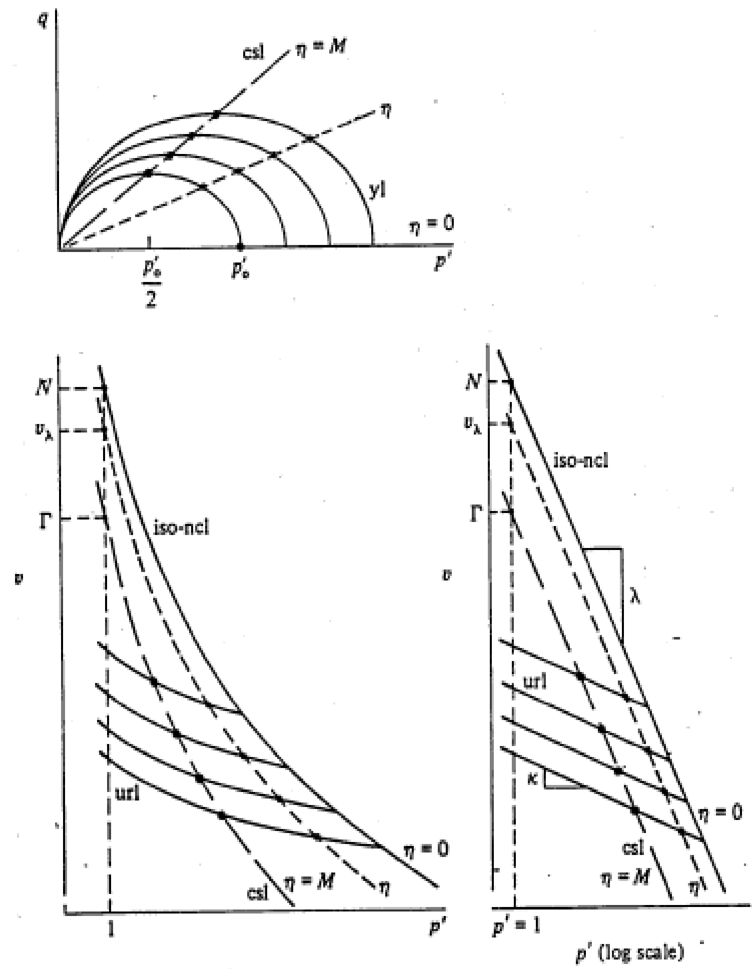
Critical state

$$NCL \quad : \qquad v = N - \lambda \, lnp'$$

$$URL \quad : \qquad v = v_k - k \, lnp'$$

$URL \quad at \ the \ particular \ \ p_0' \ :$

$$v = N - \ \lambda \, lnp_0' \ + \ \ k ln \frac{p_0'}{p'}$$



Critical state

$$at \quad p' = p'_{cs} = \frac{p'_0}{2} \quad \rightarrow \quad p'_0 = 2p'_{cs}$$

$$\therefore \quad v_{cs} = N - \lambda\, ln2\, p'_{cs} + k\, ln2$$

$$or \quad v_{cs} = N - (\lambda - k)\, ln2 - \lambda\, lnp'_{cs}$$

$$but \quad v_{cs} = \Gamma - \lambda\, lnp'_{cs}$$

$$\boxed{\Gamma = N - (\lambda - k)\, ln2}$$

Critical state

$(\lambda - k)\, ln2 \quad from \quad NCL \quad :$

This is a line in compression plane at constant vertical separation

$\Gamma \quad is \quad v \quad at \quad p'_{cs} = 1$

$(values \ \ of \ \ N \ \ and \ \ \Gamma \ \ depend \ \ on \ \ unit \ \ of \ \ stress \ \ kPa)$

Complete definition of critical state line :

$$q_{cs} = M\, p'_{cs}$$

$$v_{cs} = \Gamma - \lambda \ln p'_{cs}$$

Critical state : " q_{cs} , p'_{cs} , v_{cs} "

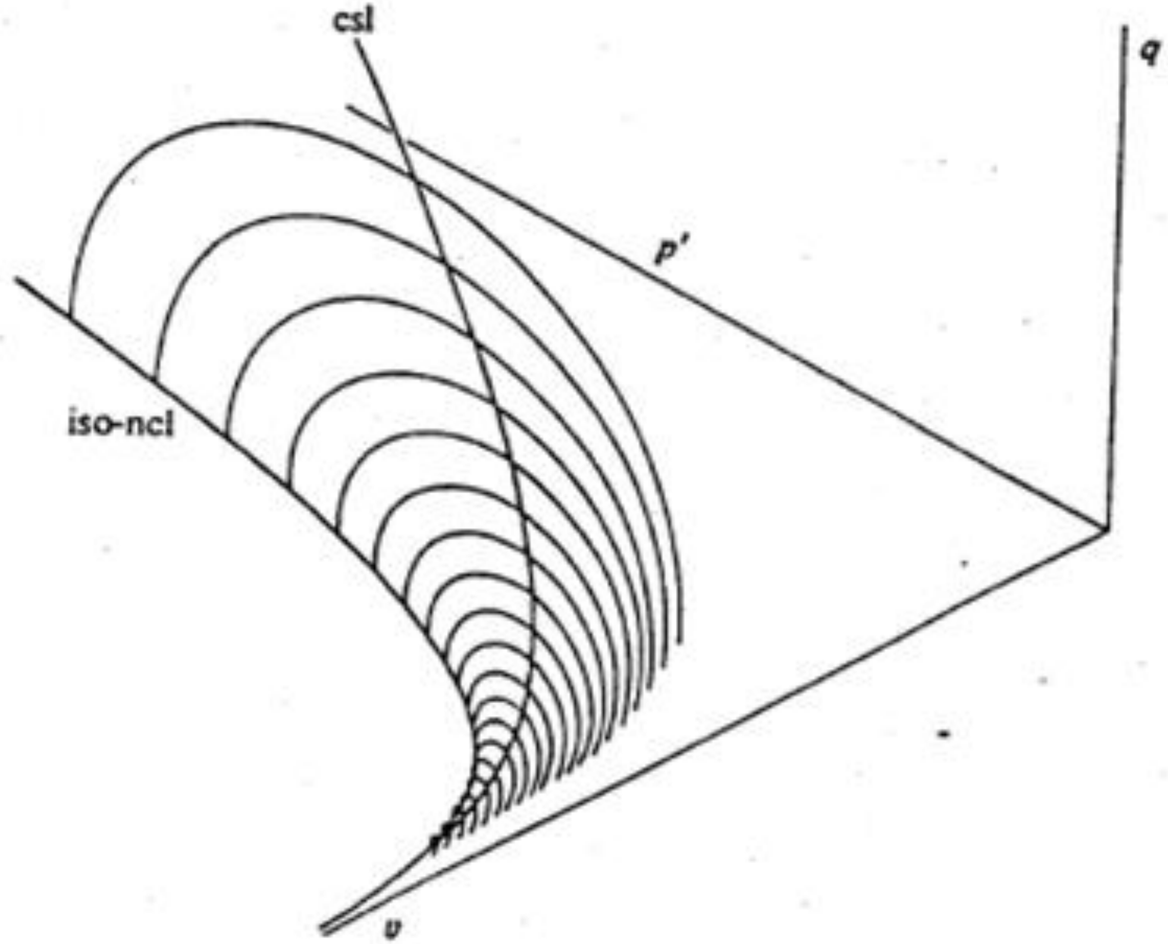
Critical state

- 3D plot :

  it is a single

  curve in

  this space .



Critical state

- 2D Representation :

for any stress ratio ; in the compression plane we have :

$$v = v_\lambda - \lambda \ lnp'$$

$$\Rightarrow \quad v_\lambda = v + \lambda \ lnp' \qquad (1)$$

Since the lines in compression plane of constant stress ratio are all parallel , it is defined by v_λ , therefore , a pair of (η , v_λ) can be used to display information about $p' : q : v$.

Critical state

- Isotropic Compression : $\eta = 0 \quad , \quad v_\lambda = N$
- At critical-state : $\eta = M \quad , \quad v_\lambda = \Gamma$

$values\ of\ v_\lambda\ for\ any\ other\ \eta$:

$$v = N - \lambda\, lnp'_0 + k\, ln\frac{p'_0}{p'} \qquad (2)$$

$From\ (1)$: $$v_\lambda = \left(N - \lambda\, lnp'_0 + k\, ln\frac{p'_0}{p'}\right) + \lambda\, lnp'$$

$$v_\lambda = N - (\lambda - k)\, ln\frac{p'_0}{p'} \qquad (3)$$

Critical state

Replace $\frac{p'_0}{p'}$ *from the yield locus equation* :

$$\frac{p'}{p'_0} = \frac{M^2}{M^2 + \eta^2}$$

$$\Rightarrow \quad v_\lambda = N - (\lambda - k)\, ln \frac{M^2 + \eta^2}{M^2}$$

$$\frac{\eta^2}{M^2} = \exp\left(\frac{N - v_\lambda}{\lambda - k}\right) - 1$$

Critical state

- This relationship is shown below :

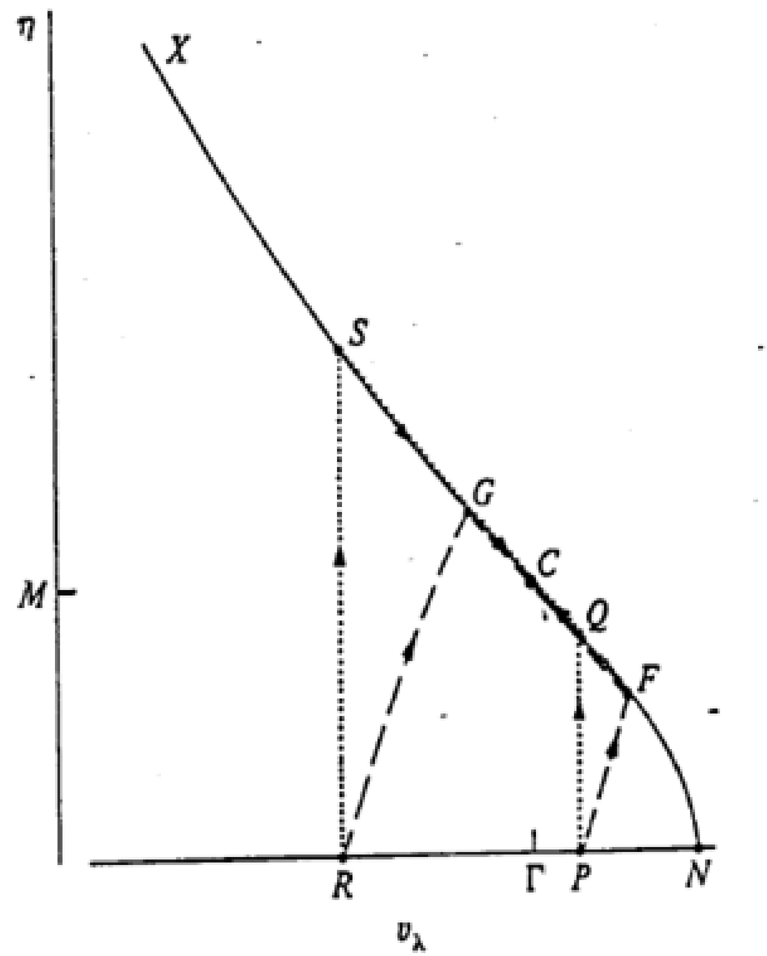
1- $\eta = 0 \quad \rightarrow \quad v_\lambda = N$

2-the "NC" path :

any normally consolidated drained and undrained test.

3-"C" critical state :

$\eta = M \quad \rightarrow \quad v_\lambda = \Gamma$



Critical state

4-Undrained test on lightly over

Consolidated starts at "p" and :

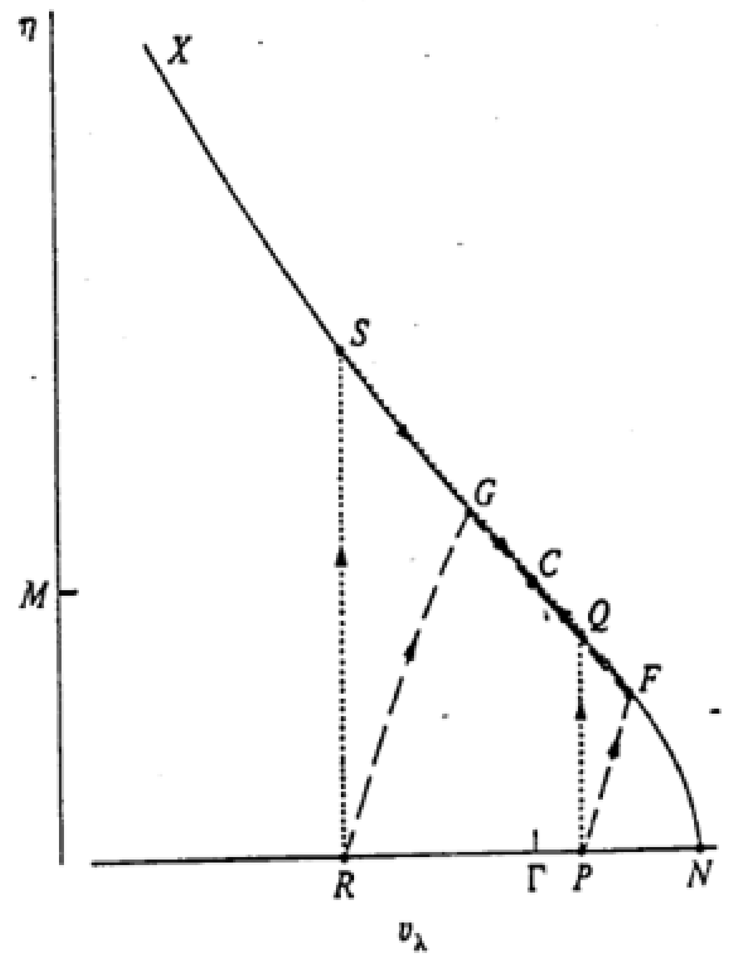
PQ , Q ⟶ C , if drained : P ⟶ F

F ⟶ C

5-heavily overconsolidated :

R ⟶ S , S ⟶ C , if drained : R ⟶ G

G ⟶ C

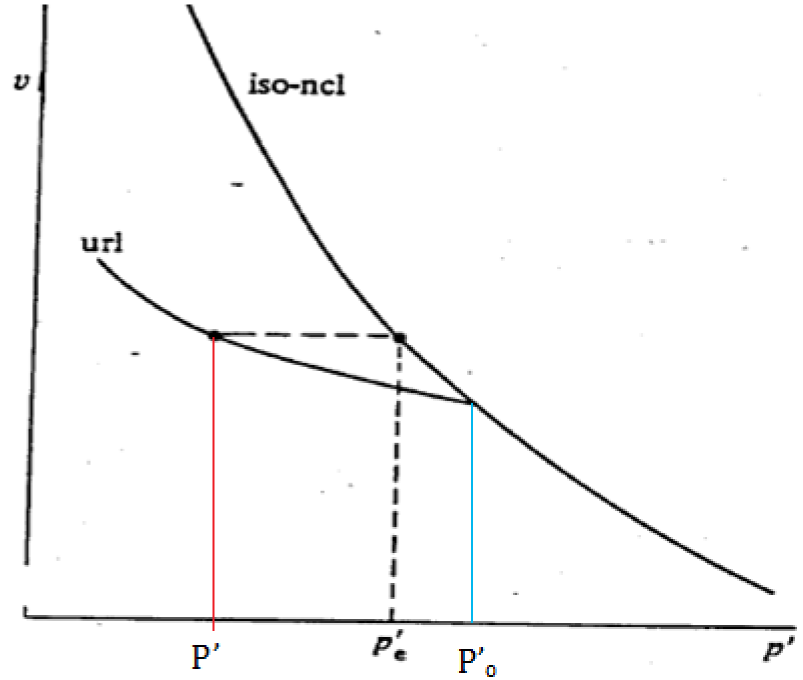


Critical state

- An alternative way :

  -Equivalent preconsolidation pressure : p'_e

> p'_e : the pressure at which in isotropic consolidation would give the current specific volume.



Critical state

Isotropic Compression line :

$$v = N - \lambda \; ln p'_e$$

$$p'_e = \exp\left(\frac{-v + N}{\lambda}\right) \qquad (1)$$

With p' inside the current yield locus of size p'_0 :

$$v = N - \lambda \; ln p'_0 \; + k \; ln \frac{p'_0}{p'} \qquad (2)$$

Critical state

$$\frac{v-N}{\lambda} = \frac{k}{\lambda}(lnp'_0 - lnp') - lnp'_0$$

$$= \frac{k-\lambda}{\lambda}\, lnp'_0 - \frac{k}{\lambda}\, lnp' \qquad (3)$$

$$(1),(3) \qquad \rightarrow \qquad \frac{p'}{p'_e} = \left(\frac{p'}{p'_0}\right)^{\Lambda}$$

$$\Lambda = \frac{\lambda - k}{\lambda}$$

Critical state

-By comparing with the yield locus :

$$\frac{p'}{p'_0} = \frac{M^2}{M^2 + \eta^2}$$

-Substitute in previous equation :

$$\Rightarrow \quad \frac{p'}{p'_e} = \left(\frac{M^2}{M^2 + \eta^2}\right)^{\Lambda} \qquad (1)$$

Equation of undrained path

-Also

$$q = \eta\, p' \quad \Rightarrow \quad \left(\frac{q}{p'_e}\right) = \eta \left(\frac{p'}{p'_e}\right) \qquad (2)$$

Critical state

(1) & (2) will give :

overconsolidated undrained :

RS ⟶ SC

PQ ⟶ QC

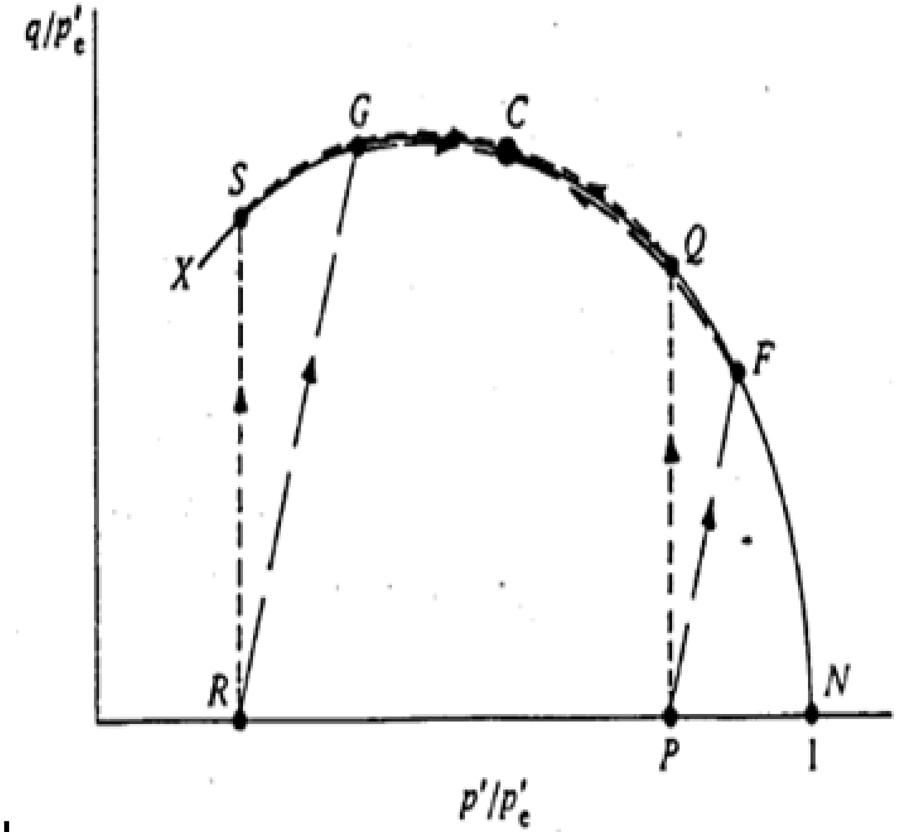
overconsolidated drained :

PF ⟶ FC

RG ⟶ GC

NC : Normally consolidated ,

both drained and undrained

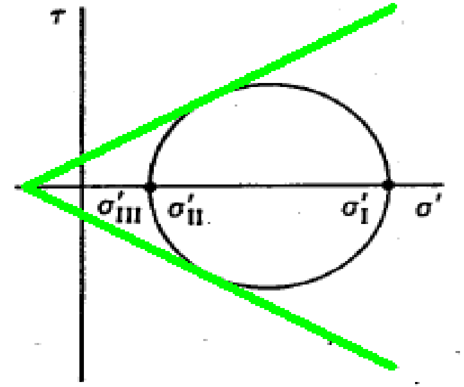


Chapter # 7

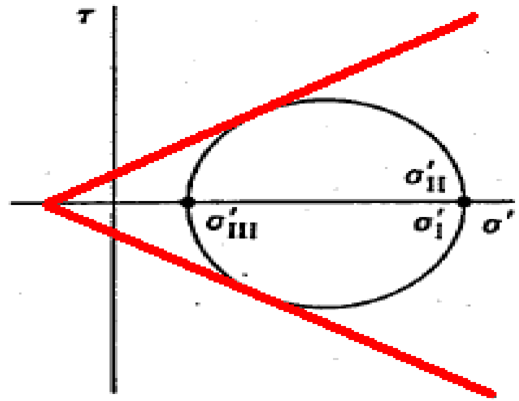
Strength of Soils

Strength of Soils

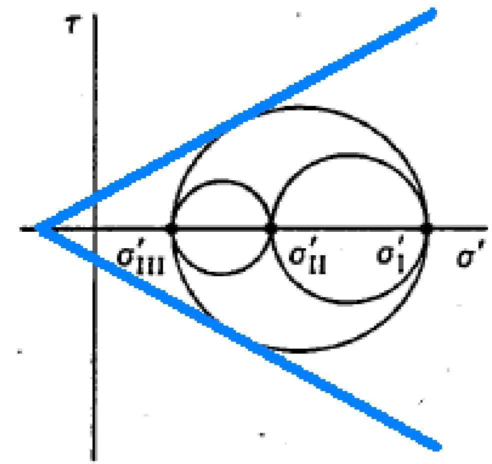
I. Triaxial Compression



II. Triaxial Extension



III. Truly Triaxial



Strength of Soils

- Mohr-Coulomb :

$$\tau = c' + \sigma' \tan \varphi' \qquad (1)$$

- In terms of principal stresses :

$$\frac{\sigma'_1 + c' \cot \varphi'}{\sigma'_3 + c' \cot \varphi'} = \frac{1 + \sin\varphi'}{1 - \sin\varphi'} \qquad (2)$$

Strength of Soils

In terms of p' & q *for triaxial compression :*

$$\sigma_2 = \sigma_3$$

$$p' = \frac{1}{3}(\sigma'_1 + 2\sigma'_3)$$

$$q = \sigma'_1 - \sigma'_3$$

$$(2) \quad \rightarrow \quad \frac{q}{p' + c' \cot \varphi'} = \frac{6 \sin\varphi'}{3 - \sin \varphi'} \quad (3)$$

Strength of Soils

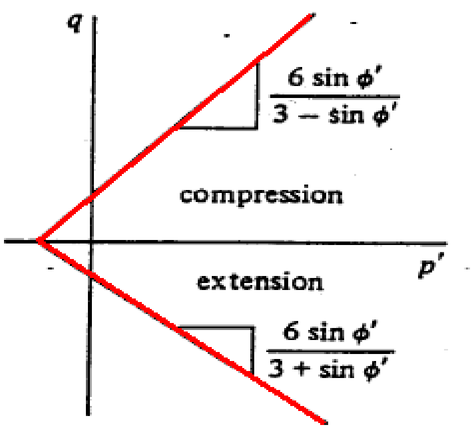
- For triaxial extension :

$$p' = \frac{2\sigma'_1 + \sigma'_3}{3} \quad ; \quad q = \sigma'_1 - \sigma'_3$$

Similarly :

$$\frac{q}{p' + c'cot\varphi'} = \frac{-6sin\varphi'}{3 + sin\varphi'} \qquad (4)$$

- Mohr-Coulomb failure criteria in q-p' space :



Strength of Soils

- Proof :

$$\frac{\sigma_1' + c'\ cot\varphi'}{2(\sigma_3 + c'\ cot\varphi)} = \frac{1 + sin\varphi'}{2(1 - sin\varphi')}$$

Combine in denominator:

$$\frac{\sigma_1' + c'\ cot\ \varphi}{\sigma_1' + c'\ cot\varphi + 2\sigma_3' + 2c'\ cot\varphi} = \frac{1 + sin\varphi'}{2 - 2sin\varphi' + 1 + sin\varphi'}$$

$$\Rightarrow \quad \frac{3[\sigma_1' + c'\ cot\varphi']}{\sigma_1' + 2\sigma_3' + 3c'cot\varphi'} = \frac{3[1 + sin\varphi]}{3 - sin\varphi}$$

Strength of Soils

- Proof (cont.)

Deduct in number:

$$\frac{3\sigma'_1 + 3c'cot\varphi' - \sigma'_1 - 2\sigma'_3 - 3c'cot\varphi'}{3(\frac{\sigma'_1 + 2\sigma'_3}{3} + c'cot\varphi')}$$

$$= \frac{3(1 + sin\varphi') - 3 + sin\varphi'}{3 - sin\varphi'}$$

$$\Rightarrow \frac{2(\sigma'_1 - \sigma'_3)}{3[\left(\frac{\sigma'_1 + 2\sigma'_3}{3}\right) + c'cot\varphi']} = \frac{4sin\varphi'}{3 - sin\varphi'}$$

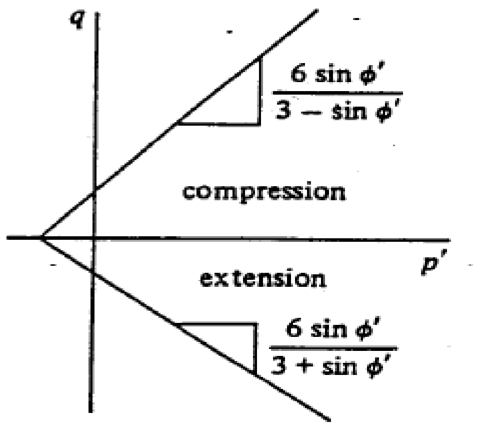
$$\Rightarrow \frac{q}{p' + c'cot\varphi'} = \frac{6sin\varphi'}{3 - sin\varphi'}$$

Strength of Soils

- In critical state

$$\eta = \frac{q}{p'} = M$$

Comparing with this figure:



soils are failing at a "purely frictional" manner at critical state. (c'=0)

Strength of Soils

- The deformation is so large that any bonding leading to cohession is destroyed. With c'=0 :

$$M = \frac{6\sin\varphi'}{3 - \sin\varphi'} \qquad \textit{Triaxial compression}$$

$$M^* = \frac{6\sin\varphi'}{3 + \sin\varphi'} \qquad \textit{Triaxial Extension}$$

- Important :

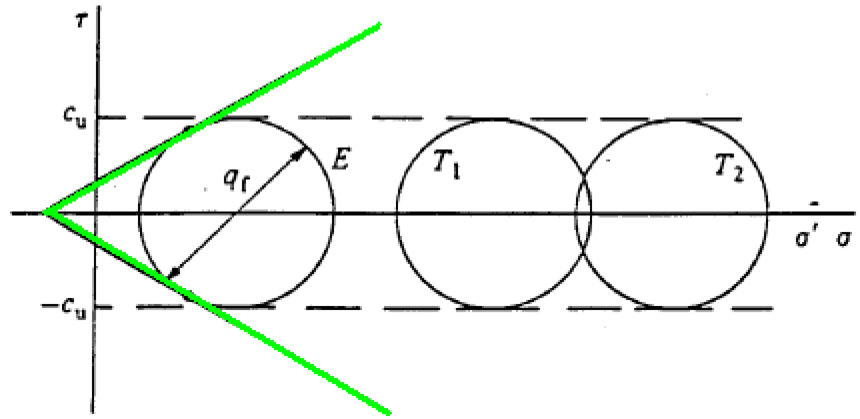
Experimental evidence ⇒ φ' is the same in triaxial compression & extension.

$$M \cong \frac{\varphi'}{25} \qquad M^* \cong \frac{\varphi'}{35}$$

Strength of Soils

Undrained Shear Strength :

$$\tau = c_u$$



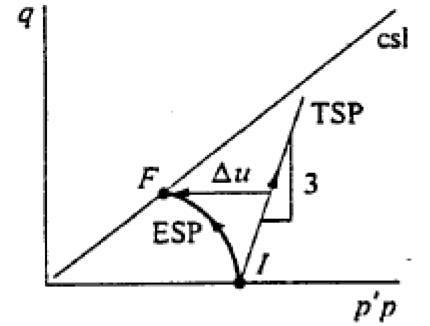
Strength of Soils

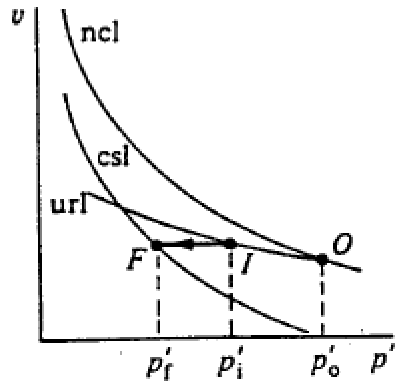
- A soil with a specific volume "v" , will end on the CSL at a mean pressure , p'_f :

$$p'_f = exp(\frac{\Gamma - v}{\lambda})$$

$$\Rightarrow \quad q_f = M p'_f$$

$$c_u = \frac{1}{2} q_f = \frac{1}{2} M p'_f = \frac{M}{2} exp(\frac{\Gamma - v}{\lambda})$$





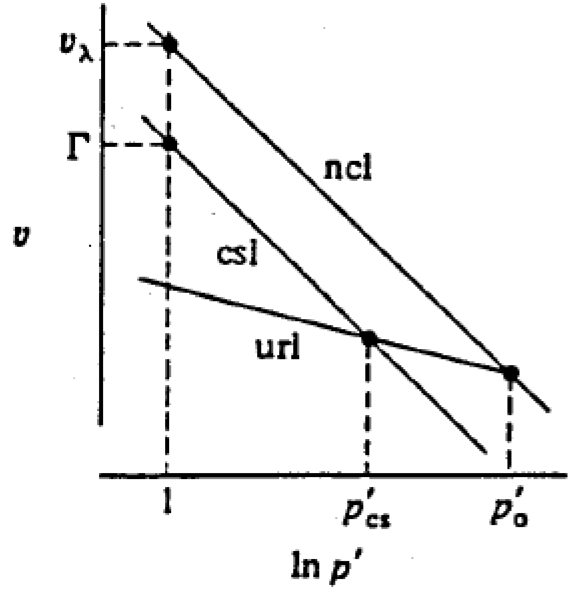
Strength of Soils

- Effect of Consolidation History :

NCL : $v = v_{\lambda} - \lambda \ln p'$ (1)

CSL : $v = \Gamma - \lambda \ln p'$ (2)

URL : $v = v_k - k \ln p'$ (3)



Separation of NCL & CSL (volume) $= v_{\lambda} - \Gamma$

This separation may be also expressed in terms of p'_0 , p'_{cs} :

$$r = \frac{p'_0}{p'_{cs}} = exp\left(\frac{v_{\lambda} - \Gamma}{\lambda - k}\right) \qquad (4)$$

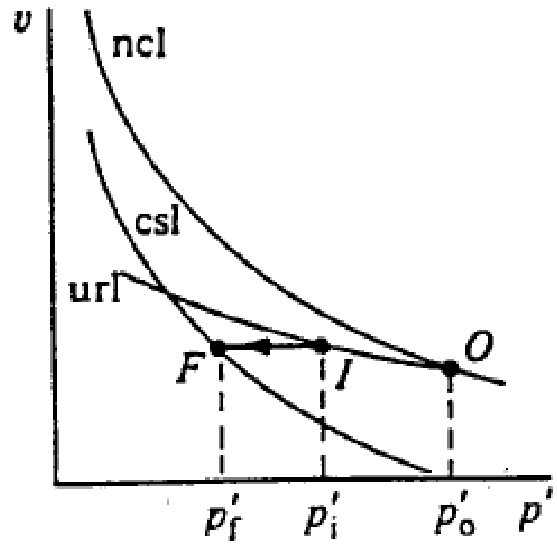
(for cam-clay model $p'_0 = 2p'_{cs} \rightarrow r = 2$)

Strength of Soils

- If a soil is NC to “O” then unloaded to “I” (not isotropically necessary)

Then , OCR , n_p is defined as :

$$n_p = \frac{p'_0}{p'_i} \qquad (4b)$$



$$And \quad v_i = v_\lambda - \lambda\, ln\, p'_0 + k\, ln\,(n_p) \qquad (5)$$

$$Also \quad v_i = \Gamma - \lambda\, ln\, p'_f \qquad (6)$$

Strength of Soils

From (5) & (6) :

$$\Gamma - \lambda \ln p'_f = v_\lambda - \lambda \ln p'_0 + k \ln (n_p)$$

$$\ln p'_f = \frac{\Gamma - v_\lambda}{\lambda} + \ln p'_0 - \frac{k}{\lambda} \ln (n_p)$$

$$Or \quad p'_f = \exp\left[\frac{\Gamma - v_\lambda}{\lambda} + \ln p'_0 - \frac{k}{\lambda} \ln (n_p)\right]$$

$$But \quad q_f = M p'_f \quad \& \quad C_u = \frac{1}{2} M p'_f$$

$$C_u = \frac{M}{2} \exp\left[\frac{\Gamma - v_\lambda}{\lambda} + \ln p'_0 - \frac{k}{\lambda} \ln (n_p)\right] \qquad (7)$$

Strength of Soils

Using (4) & (4b) with (7) :

$$C_u = p_i' \frac{M}{2} \left(\frac{n_p}{r} \right)^{\Lambda} \quad ; \quad \Lambda = \frac{\lambda - k}{\lambda} \qquad (8)$$

Note :

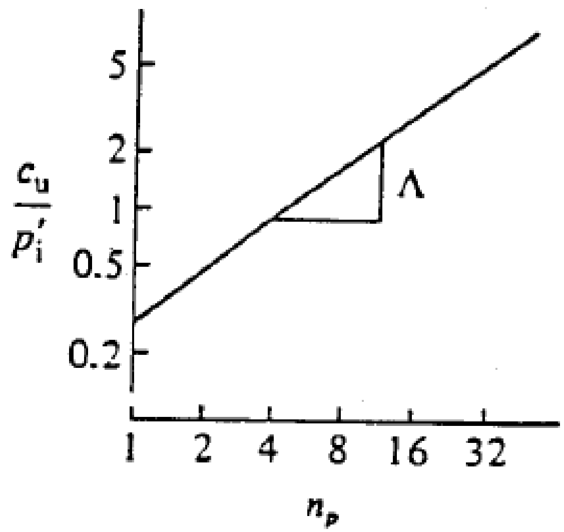
The equation links the total undrained shear strength of soils with effective stress parameters M , k ,λ , ... and the consolidation history of the soil.

Strength of Soils

From (8) :

$$\left(\frac{C_u}{p_i}\right)_{nc} = \frac{M}{2}\left(\frac{1}{r}\right)^{\Lambda} = \frac{M}{2}\, r^{-\Lambda} \qquad (9)$$

$$\frac{\left(\frac{C_u}{p_i}\right)}{\left(\frac{C_u}{P_i}\right)_{nc}} = {n_p}^{\Lambda}$$



Strength of Soils

In terms of normal consolidation instead of isotropic consolidation :

Define : $n = \frac{\sigma'_{v0}}{\sigma'_{vi}}$

Then : $$\frac{\left(\frac{C_u}{\sigma'_{vi}}\right)}{\left(\frac{C_u}{\sigma'_{vi}}\right)_{nc}} = \left(\frac{n}{n_p}\right)\left(n_p\right)^{\Lambda} \qquad (10)$$

From fig. (7.8b) $$\frac{\left(\frac{C_u}{\sigma'_{vi}}\right)}{\left(\frac{C_u}{\sigma'_{vi}}\right)_{nc}} = n^N \qquad N \cong 0.8$$

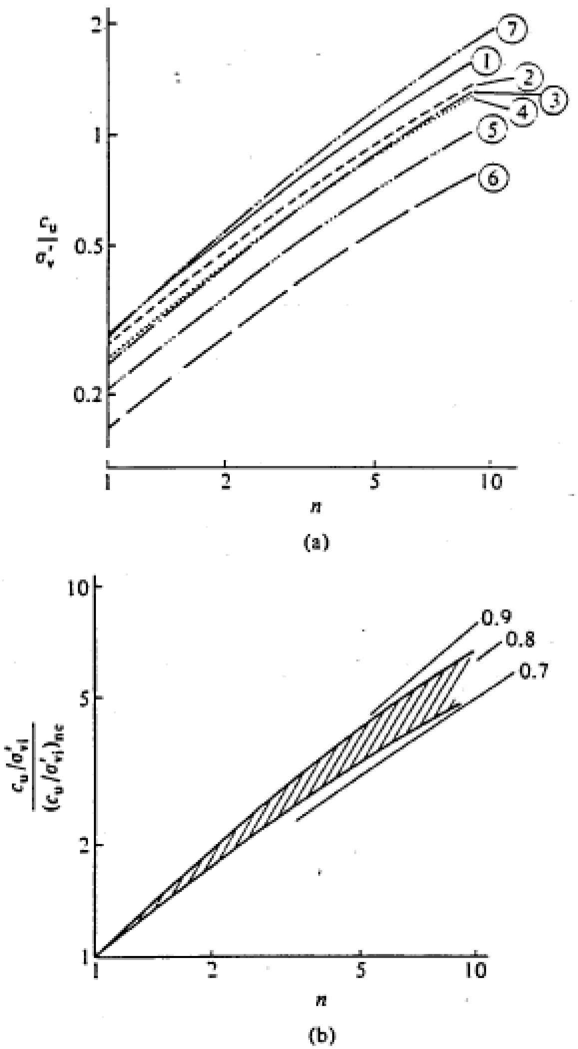


Fig. 7.8 Ratio of undrained strength to initial vertical effective stress (c_u/σ'_{vi}) varying with overconsolidation ratio n: (1) Drammen clay ($I_p = 0.30$) (after Andresen, Berre, Kleven, and Lunne, 1979); (2) Maine organic clay ($I_p = 0.34$); (3) Bangkok clay ($I_p = 0.41$); (4) Atchafalaya clay ($I_p = 0.75$); (5) AGS CH clay ($I_p = 0.41$); (6) Boston blue clay ($I_p = 0.21$); (7) Connecticut Valley varved clay ($I_p = 0.39/0.12$) (after Ladd, 1981).

Strength of Soils

- Critical state line V pore water pressure at failure :

Previously : $$\delta u = \delta p + a\,\delta q \qquad (1)$$

a : current slope of the undrained effective stress path

"a" is not a soil constant but depends on the current stress-state and history of consolidation.

an average value : $$a_f = \frac{-\Delta p'}{\Delta q} \qquad (2)$$

Remember :

$$n_p = \frac{p'_0}{p'_i} \quad ; \quad q_f = M\, p'_i \left(\frac{n_p}{r}\right)^{\Lambda} \qquad (3)$$

Strength of Soils

-In triaxial compression test :

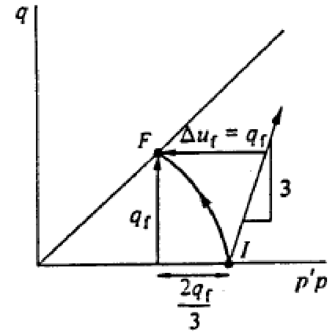
$$\Delta q = q_f \quad ; \qquad \Delta p = \frac{1}{3}\Delta q = \frac{1}{3}\, q_f \qquad (4)$$

the effective mean stress at the end of test :

$$p'_f = \frac{q_f}{M} = p'_i\left(\frac{n_p}{r}\right)^{\Lambda} \qquad (5)$$

the pore water pressure at failure (from 1 & 2) :

$$\Delta u = \Delta p + a_f * \Delta q$$



Strength of Soils

$$\Delta u = \Delta p + a_f * \Delta q$$

$$= \Delta p - \frac{\Delta p'}{\Delta q} * \Delta q$$

$$= \Delta p - \Delta p'$$

$$= \Delta p - (p'_f - p'_I)$$

$$= \Delta p - p'_f + p'_I$$

$$= p'_I - \frac{q_f}{M} + \frac{1}{3} q_f \quad \leftarrow \quad from\,(4)$$

Substitute from (5) :

$$\Delta u = p'_I \left[1 + M \left(\frac{n_p}{r} \right)^{\Lambda} \left(\frac{1}{3} - \frac{1}{M} \right) \right] \qquad (6)$$

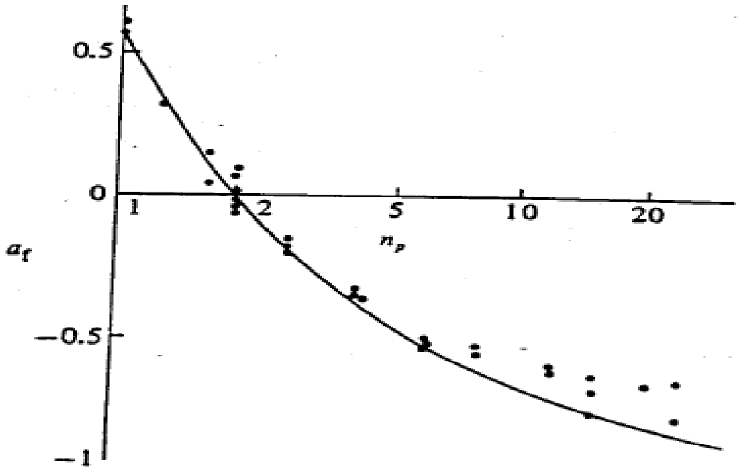
Strength of Soils

But $$a_f = -\frac{p'_f - p'_I}{q_f} \qquad (7)$$

Replace from (5) in (7) :

$$a_f = \frac{1}{M}\left[\left(\frac{n_p}{r}\right)^{-\Lambda} - 1\right] \qquad (8)$$

Dependence of pore pressure parameter a_f at failure on overconsolidation ratio for isotropically overconsolidated Weald clay (after Bishop and Henkel, 1957).



Strength of Soils

- For normally consolidated clays :

$$M \cong 0.95 - 1.0$$

$$n_p = 1 \quad ; \quad r = 2$$

$$\frac{k}{\lambda} \cong \frac{1}{5} \ to \ \frac{1}{10} \qquad \left(take \ \frac{1}{5}\right)$$

$$\Lambda = \frac{\lambda - k}{\lambda} = 1 - \frac{k}{\lambda} = 0.8$$

From (8) $\rightarrow$ $$a_f \cong 0.95 \left[\left(\frac{1}{2}\right)^{-0.8} - 1\right] = 0.7$$

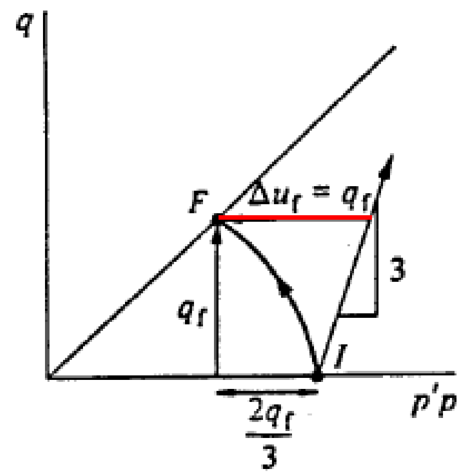
$$a_f \cong 0.6 \ to \ 0.7 \qquad for \ most \ NC \ clays$$

Strength of Soils

Taking $a_f \cong \dfrac{2}{3}$

$$\Delta u \cong \left(\frac{2}{3} + \frac{1}{3}\right) q_f$$

$$\Delta u \cong q_f$$



For n_p*=2 & based on cam-clay model (r=2)*

$$a_f = 0 \quad ; \quad (\textit{since the elastic behavior } \Delta p' = 0)$$

Strength of Soils

- Peak Strength :

the locus of peak points consists of two sections :

1-Lightly "OC" "NC" samples

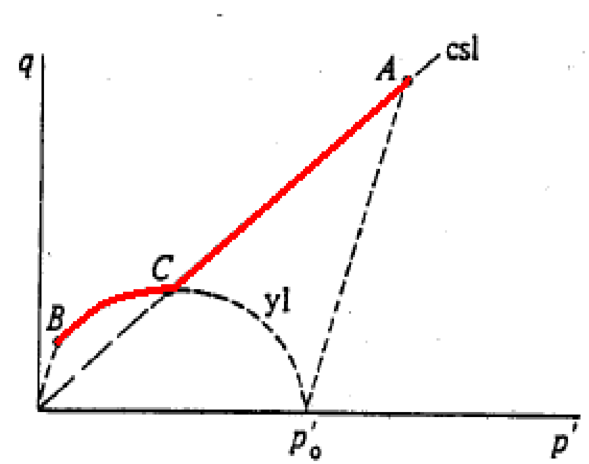
(OCR ≤ 2)

CSL between A C is

the peak deviatoric stress.

2-Heavily "OC" samples between B C

(drained triaxial test)



"point B for a sample at zero cell pressure"

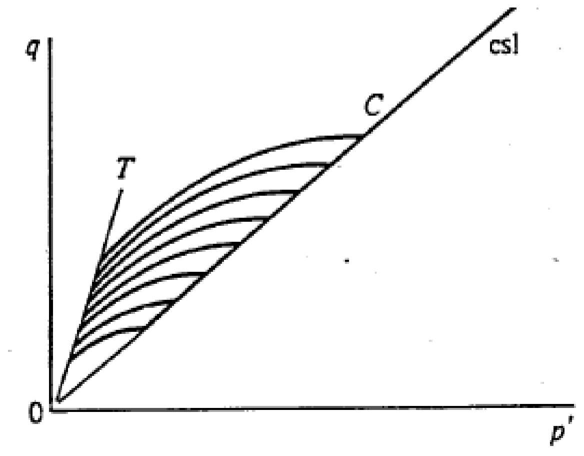
Strength of Soils

- Note :

It is inappropriate to get asingle Mohr-coulomb strength equation to this set of data.

with similar test with different past max consolidation pressure , we get :

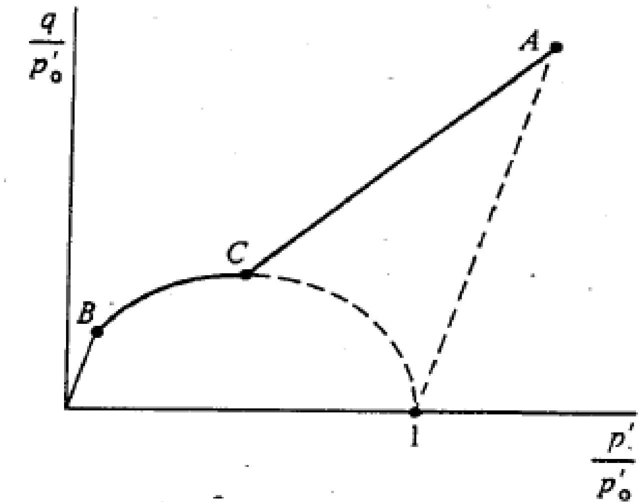
Peak is the zero TOC for heavily OC soils under drained triaxial compression.



[No single Mohr-Coulomb line]

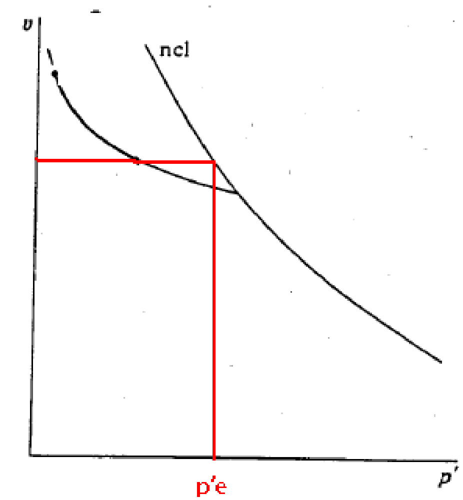
Strength of Soils

To make this curve into a single representation , by normalization of the axes :



It is necessary to deduce the max past consolidation pressure , but having the saturated water content

→ v (specific volume)

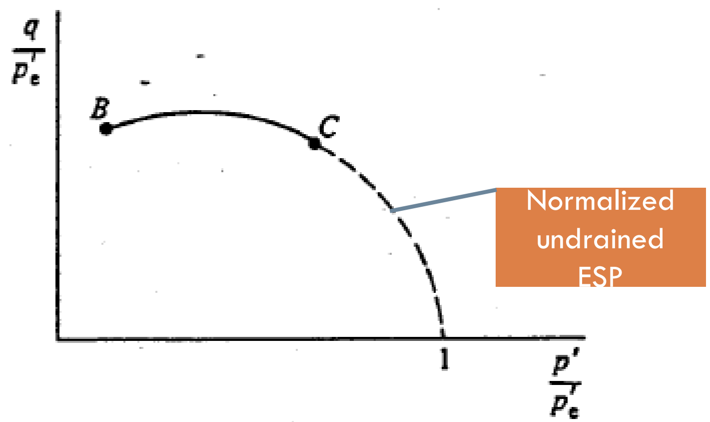


And the equivalent consolidation pressure p'_e May be deduced.(p'_e)

Strength of Soils

- As explained previously the normalization with gives a unique curve "BC" :

The CSL become a point "C"



Chapter # 8

Hyperbolic Model

Hyperbolic Model

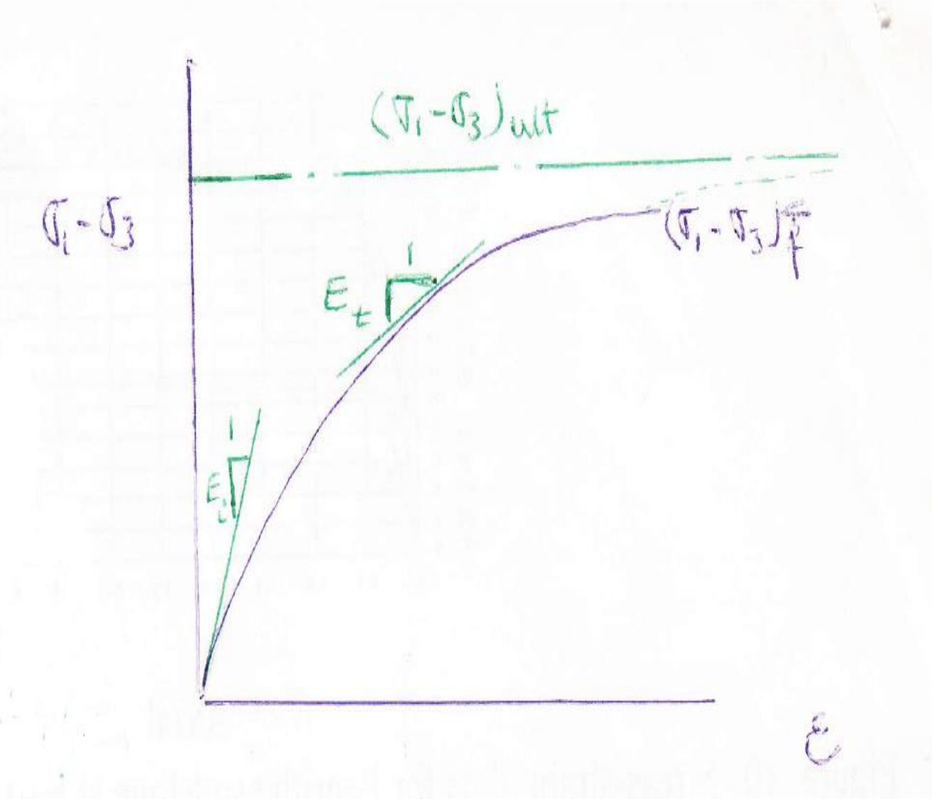
a) Loading :

$$(\sigma_1 - \sigma_3) = \frac{\varepsilon}{\frac{1}{E_i} + \frac{\varepsilon}{(\sigma_1 - \sigma_3)_{ult}}}$$

$$R_f = \frac{(\sigma_1 - \sigma_3)_f}{(\sigma_1 - \sigma_3)_{ult}}$$

$$0.6 \leq R_f \leq 0.9$$

$$(\sigma_1 - \sigma_3)_f = \frac{2c\cos\varphi + 2\sigma_3 \sin\varphi}{1 - \sin\varphi}$$



Hyperbolic Model

$$E_t = \frac{d(\sigma_1 - \sigma_3)}{d\varepsilon} \quad \Rightarrow \quad E_t = [1 - R_f(SL)]^2 \, . E_i$$

$where:$

$$E_i = K\, p_a \left(\frac{\sigma_3}{p_a}\right)^n \qquad SL = \frac{\sigma_1 - \sigma_3}{(\sigma_1 - \sigma_3)_f}$$

E_t *is a function of* :

$-$*stress level*

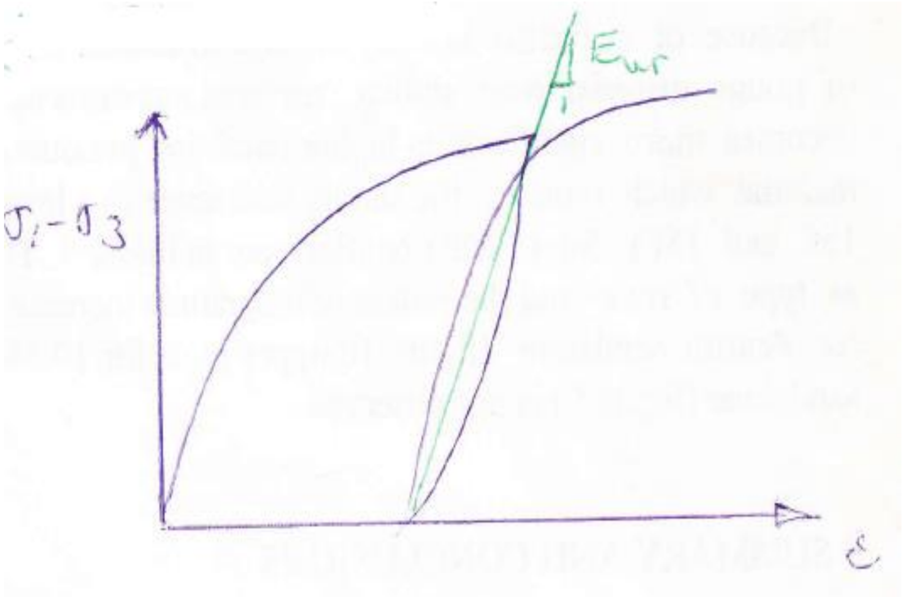
$-$*confining pressure* (σ_3)

Hyperbolic Model

b) Unloading – Reloading :

$$E_{ur} = K_{ur}\, p_a \left(\frac{\sigma_3}{p_a}\right)^n$$

$$K_{ur} \cong (1.2-3)K$$

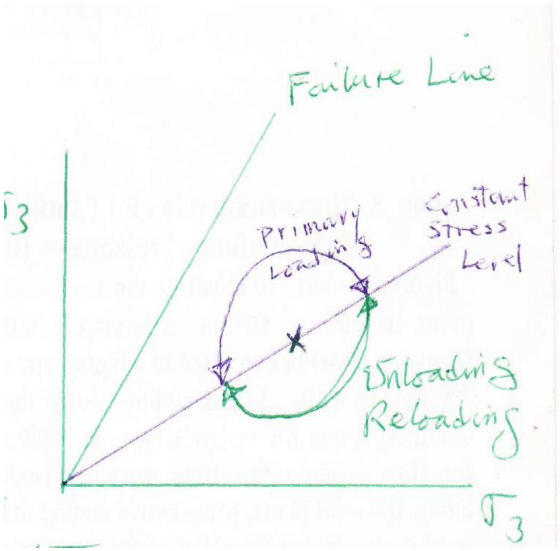


$$SL = (\sigma_1 - \sigma_3)/(\sigma_1 - \sigma_3)_f$$

$If \quad SL < SL_{max.past}$ → $Unloading - Reloading$

Hyperbolic Model

$$Stress - state \qquad SS = SL\sqrt[4]{\frac{\sigma_3}{p_a}}$$



- $Unloading\ Reloading \quad if \quad SS < SS_{max\,.\,past}$

- $(SL)_{critical}$ $above\ which\ primary\ loading\ is\ assumed$

$$(SL)_{critical} = \frac{SS_{max.\ past}}{\sqrt[4]{\frac{\sigma_3}{p_a}}}$$

Hyperbolic Model

- Bulk Modulus :

$$B = K_b \, p_a \left(\frac{\sigma_3}{p_a}\right)^m$$

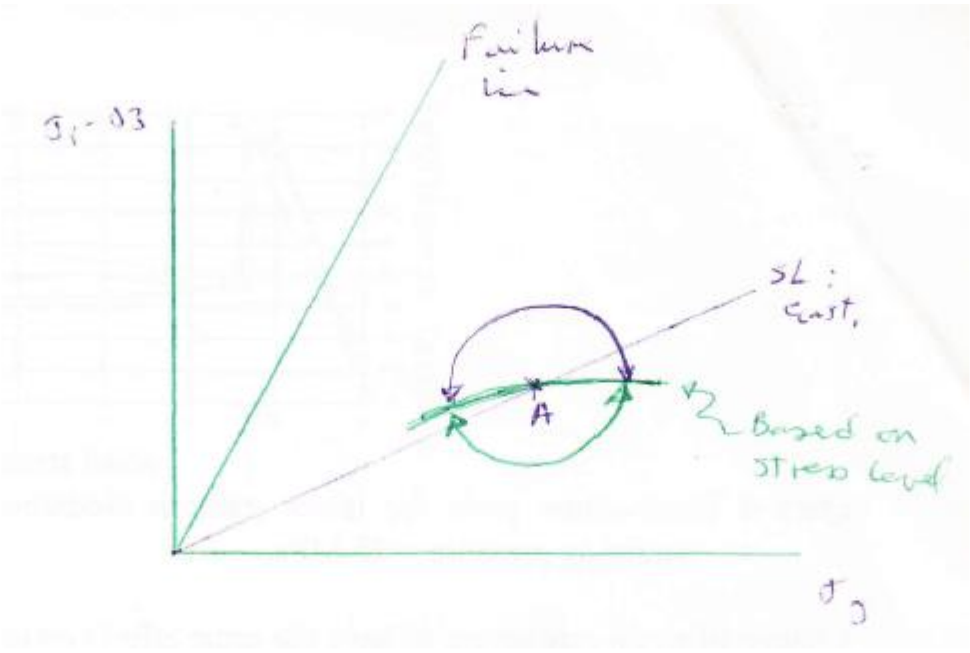
Hence : $\nu = \dfrac{1}{2} - \dfrac{E_t}{6B}$

$$\begin{Bmatrix} \Delta\sigma_x \\ \Delta\sigma_y \\ \Delta\tau_{xy} \end{Bmatrix} = \frac{3B}{9B - E_t} \begin{bmatrix} 3B + E_t & 3B - E_t & 0 \\ 3B - E_t & 3B + E_t & 0 \\ 0 & 0 & E_t \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_x \\ \Delta\varepsilon_y \\ \Delta\gamma_{xy} \end{Bmatrix}$$

$\nu < 0.5$ $\quad$ $\nu = 0.5 \rightarrow$ *Numerical Instability*

→ $E_t \;,\; B \;,\; E_{ur}$

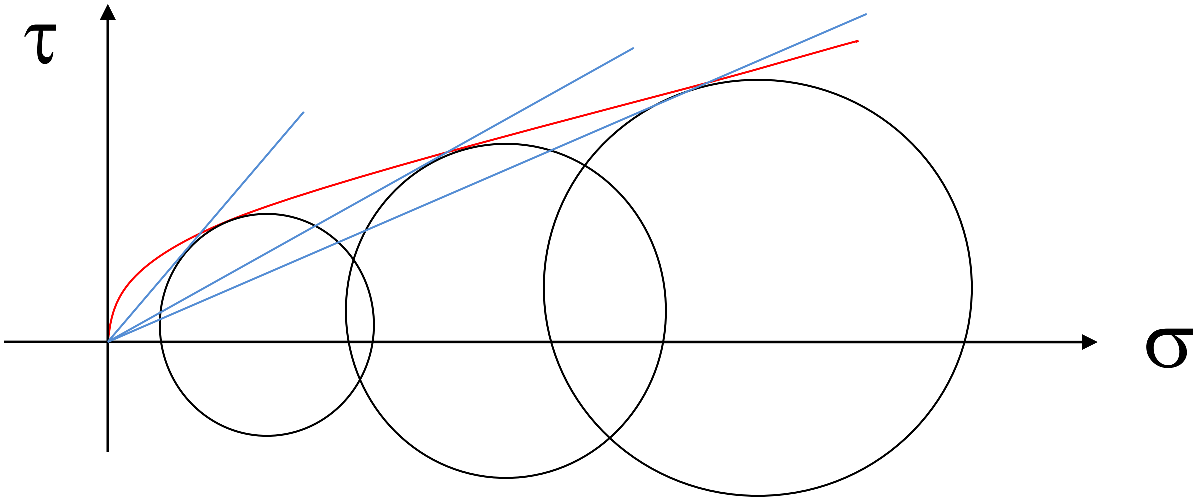
Hyperbolic Model



Hyperbolic Model

Nonlinear Failure envelope:

$$\varphi = \varphi_0 - \Delta\varphi \; log(\frac{\sigma_3}{p_a})$$



Hyperbolic Model

- Evaluation of Hyperbolic Parameters :

a) Evaluation of K & n :

$$E_i = K\, p_a \left(\frac{\sigma_3}{p_a}\right)^n$$

Determine E_i for each test , plot E_i versus σ_3 on log scales.

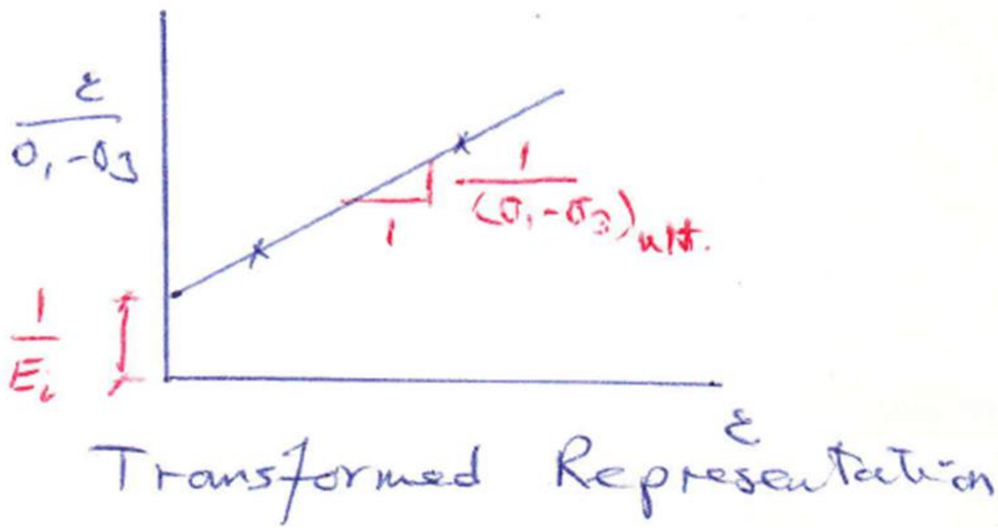
For determining E_i :

hyperbolic model : $$\sigma_1 - \sigma_3 = \frac{\varepsilon}{\frac{1}{E_i} + \frac{\varepsilon}{(\sigma_1 - \sigma_3)_{ult}}}$$

$$\Rightarrow \quad \frac{\varepsilon}{\sigma_1 - \sigma_3} = \frac{1}{E_i} + \frac{\varepsilon}{(\sigma_1 - \sigma_3)_{ult}}$$

Hyperbolic Model

- $\frac{\varepsilon}{\sigma_1 - \sigma_3}$ *versus* ε *is line*



Hyperbolic Model

- The data plotted on the transformed section only for 70% and 95% of strength (Duncan & Chang believe this gives the best fit for the hyperbola).

Only two points appear on the transformed section .

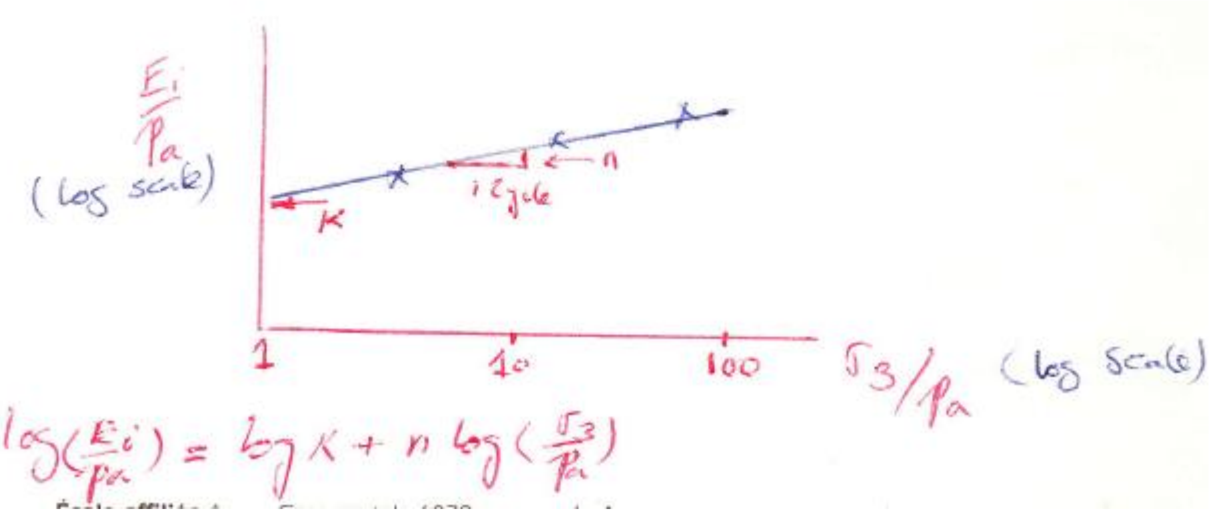
$$E_i \; , \; (\sigma_1 - \sigma_3)_{ult} \; , \quad \sigma_{3_1}$$

$$E_i \; , \; (\sigma_1 - \sigma_3)_{ult} \; , \quad \sigma_{3_2}$$

$$E_i \; , \; (\sigma_1 - \sigma_3)_{ult} \; , \quad \sigma_{3_3}$$

Hyperbolic Model

- $Next\ a\ plot\ of\ \frac{E_i}{p_a}\ versus\ \frac{\sigma_3}{p_a}$:



Hyperbolic Model

b) Evaluation of $\boldsymbol{R_f}$:

For the 3 $(\sigma_1 - \sigma_3)_{ult}$ obtained , the values of $(\sigma_1 - \sigma_3)_{failure}$ are determined correspondingly , then :

$$(R_f)_{Ave} = \frac{1}{N} \sum \frac{(\sigma_1 - \sigma_3)_f}{(\sigma_1 - \sigma_3)_{ult}}$$

N : No. of test

c) C & $\boldsymbol{\varphi}$:

Usual procedure by fitting Mohr-Coulomb line .

Hyperbolic Model

d) $\Delta\varphi$

usually for cohesionless soils , the curvature is higher , and single φ for different σ_3 in the dam body is difficult (like rockfill material).

→ φ is determined for each Mohr circle with zero cohesion :

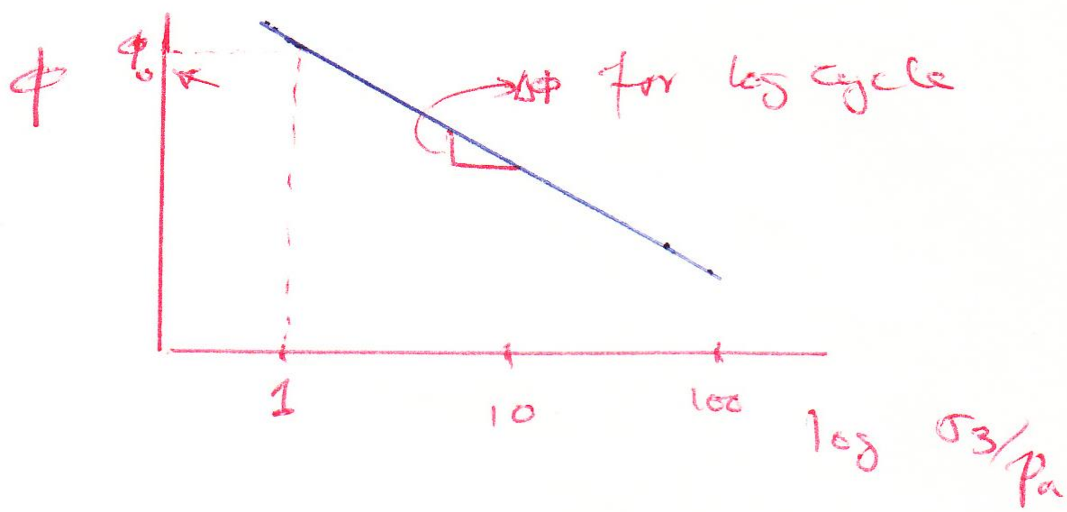
→

$$\varphi = sin^{-1}\left(\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}\right)$$

Hyperbolic Model

By drawing φ versus log of $\frac{\sigma_3}{p_a}$ and fitting a line :

$$\varphi = \varphi_0 - \Delta\varphi \ \ log_{10}(\frac{\sigma_3}{p_a})$$



Hyperbolic Model

e) Evaluation of K_{ur} :

$$K_{ur} = \frac{E_{ur}}{p_a \left(\frac{\sigma_3}{p_a}\right)^n}$$

n : determined previously

On unknown , K_{ur} , may be determined from a single unloading curve , and best value of

usually $1.2 \leq \frac{K_{ur}}{K} \leq 3$

Hyperbolic Model

f) Evaluation of K_b & m :

$$B = K_b \, p_a \left(\frac{\sigma_3}{p_a}\right)^m$$

Two steps :

$-$*Determined B for each test .*

$-$*Plot B versus* σ_3 *on log – log scales.*

for soils where volume change curves do not reach horizontal tangent prior to the stage at which 70% of strength is mobilized , B is calculated at 70% strength level . Otherwise , where the tangent is horizontal .

Hyperbolic Model

$$B = \frac{\Delta(mean\ pressure)}{\varepsilon_v}$$

$$= \frac{\Delta(\sigma_1 + \sigma_2 + \sigma_3)}{3} \frac{1}{\varepsilon_v}$$

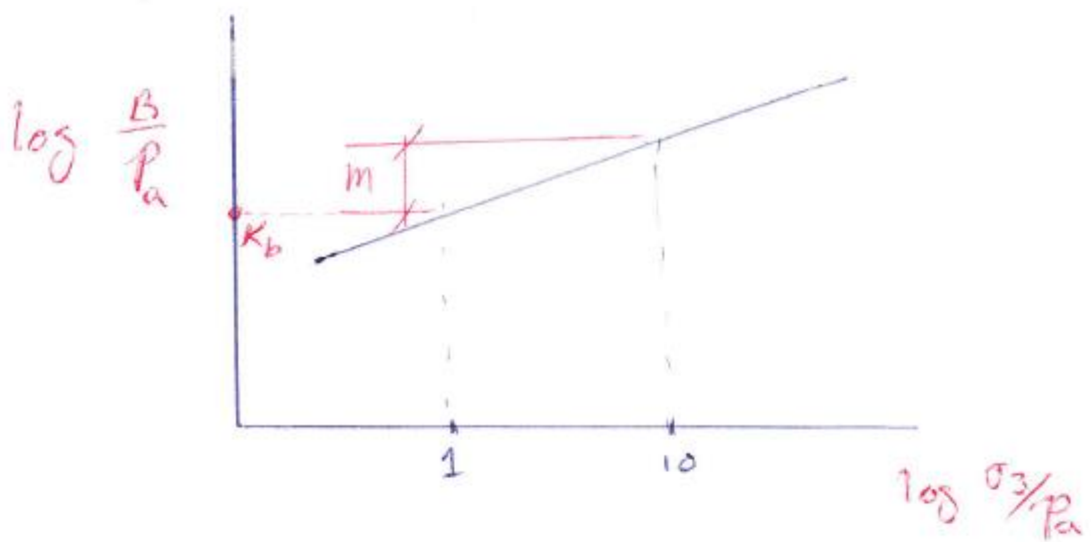
$$\Delta\sigma_2 = \Delta\sigma_3 = 0$$

$$\Delta\sigma_1 = \Delta\left(\frac{P}{A}\right) = (\sigma_1 - \sigma_3)$$

$$\sigma_1 = \sigma_3 + \frac{P}{A}$$

$$B = \frac{\sigma_1 - \sigma_3}{3\varepsilon_v}$$

Hyperbolic Model



$$0 \leq \nu \leq 0.5 \qquad or \qquad 0 \leq \nu \leq 0.49$$

$$Restrict \quad \frac{1}{3} E_t \leq B \leq 17\, E_t$$